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# A Verified SAT Solver with Watched Literals Using Imperative HOL (Extended Abstract)

Mathias Fleury

Jasmin C. Blanchette

Peter Lammich

VirginiaTech
Invent the Future

SAARLAND
UNIVERSITY
SAARBRÜCKEN
GRADUATE SCHOOL OF
COMPUTER SCIENCE

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# How reliable are SAT solvers?

Two ways to ensure correctness:

- certify the certificate
  - certificates are huge
- verification of the code
  - code will not be competitive
  - allows to study metatheory
  - useful if non-checkable techniques are required

# How reliable are SAT solvers?

Two ways to ensure correctness:

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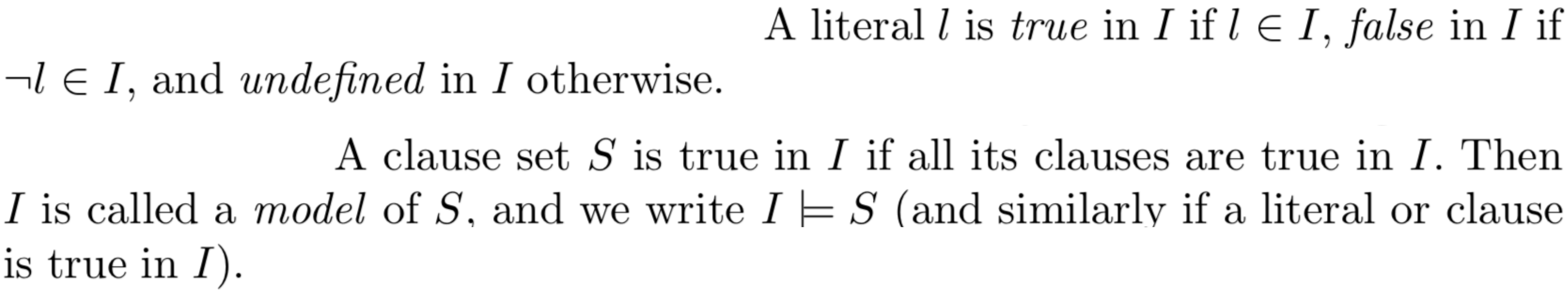
  - certificates are huge
- verification of the code
  - code will not be competitive
  - allows to study metatheory

# How reliable is the theory?

Conference version

Branch and Bound for Boolean Optimization and the Generation of Optimality Certificates

Javier Larrosa, Robert Nieuwenhuis, Albert Oliveras, and Enric Rodríguez-Carbonell (SAT 2009)



A literal $l$ is *true* in $I$ if $l \in I$, *false* in $I$ if $\neg l \in I$, and *undefined* in $I$ otherwise.

A clause set $S$ is true in $I$ if all its clauses are true in $I$. Then $I$ is called a *model* of $S$, and we write $I \models S$ (and similarly if a literal or clause is true in $I$).

# How reliable is the theory?

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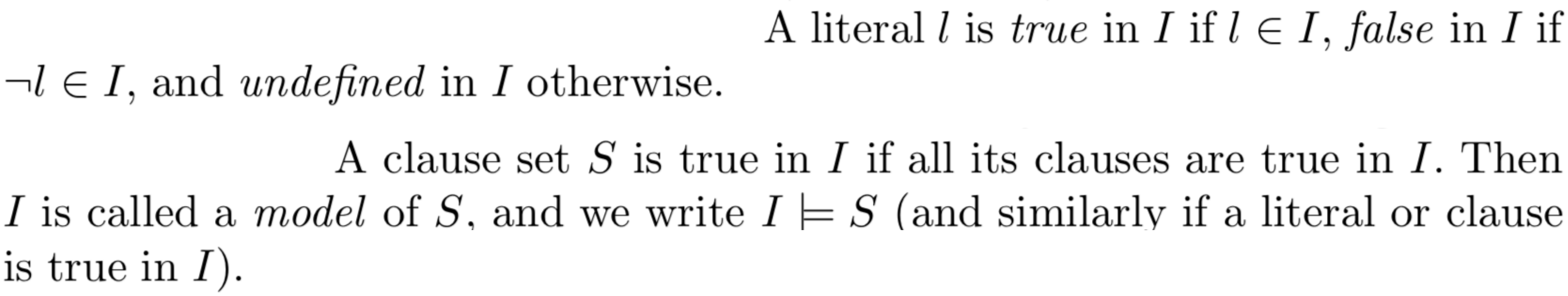

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A clause set $S$ is true in $I$ if all its clauses are true in $I$. Then $I$ is called a *model* of $S$, and we write $I \models S$ (and similarly if a literal or clause is true in $I$).

Journal version

A Framework for Certified Boolean Branch-and-Bound Optimization
Javier Larrosa, Robert Nieuwenhuis, Albert Oliveras, and Enric Rodríguez-Carbonell (JAR 2011)

literals of a clause $C$ are false in $I$. A clause set $S$ is true in $I$ if all its clauses are true in $I$; if $I$ is also total, then $I$ is called a *total model* of $S$, and we write $I \models S$.

**I certify your proof**

Isabelle

IsaFoL

# IsaFoL project

Isabelle Formalisation of Logic

# IsaFoL

- FO resolution
  by Schlichtkrull (ITP 2016)

- CDCL with learn, forget, restart, and incrementality
  by Blanchette, Fleury, Weidenbach (IJCAR 2016)

- GRAT certificate checker
  by Lammich (CADE-26, 2017)

- A verified SAT solver with watched literals
  by Fleury, Blanchette, Lammich (CPP 2018, now)

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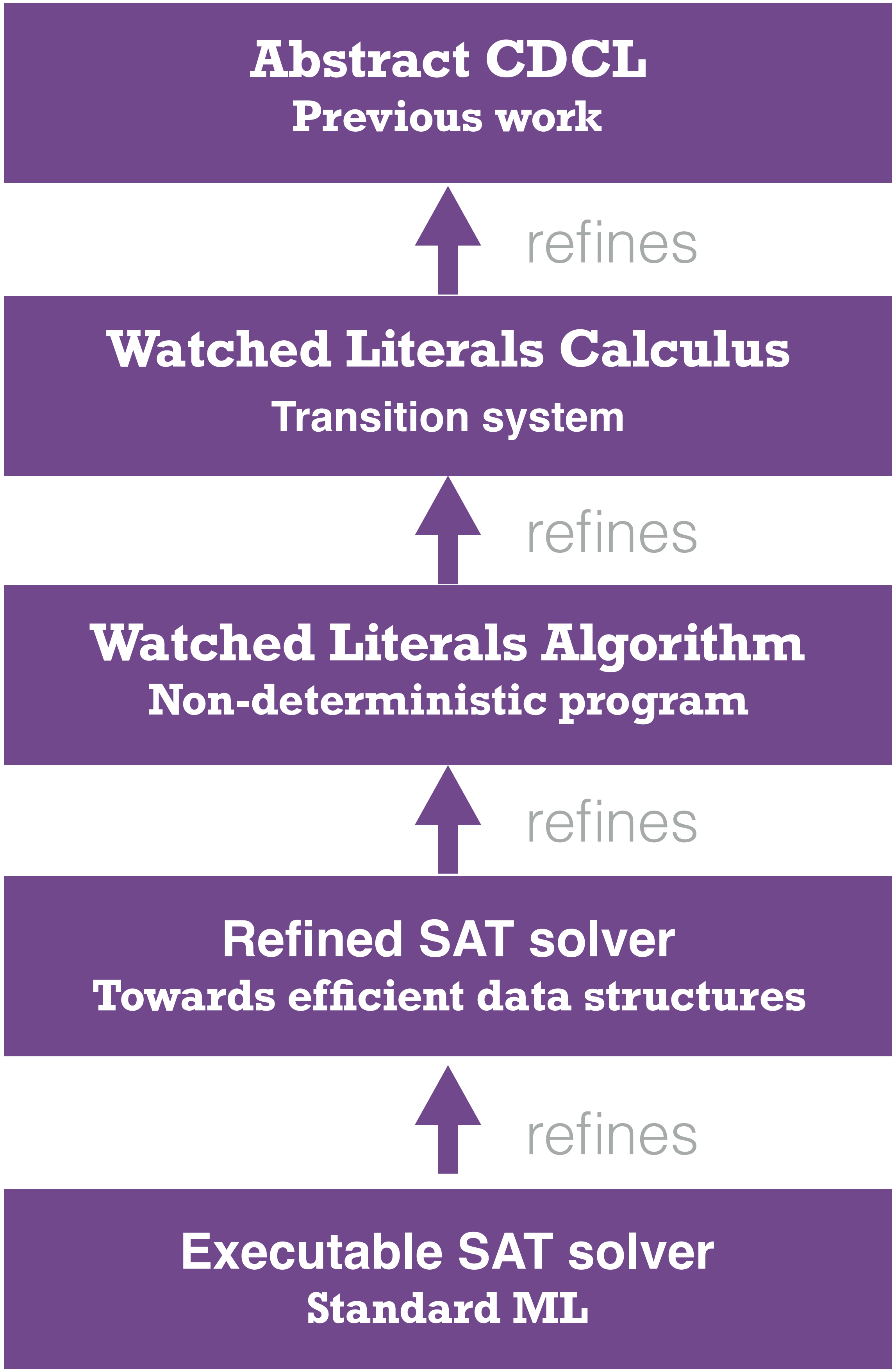

**Abstract CDCL**
**Previous work**

refines

**Watched Literals Calculus**
**Transition system**

refines

**Watched Literals Algorithm**
**Non-deterministic program**

refines

**Refined SAT solver**
**Towards efficient data structures**

refines

**Executable SAT solver**
**Standard ML**

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# Abstract CDCL

## Previous work

in Isabelle

## Propagate rule

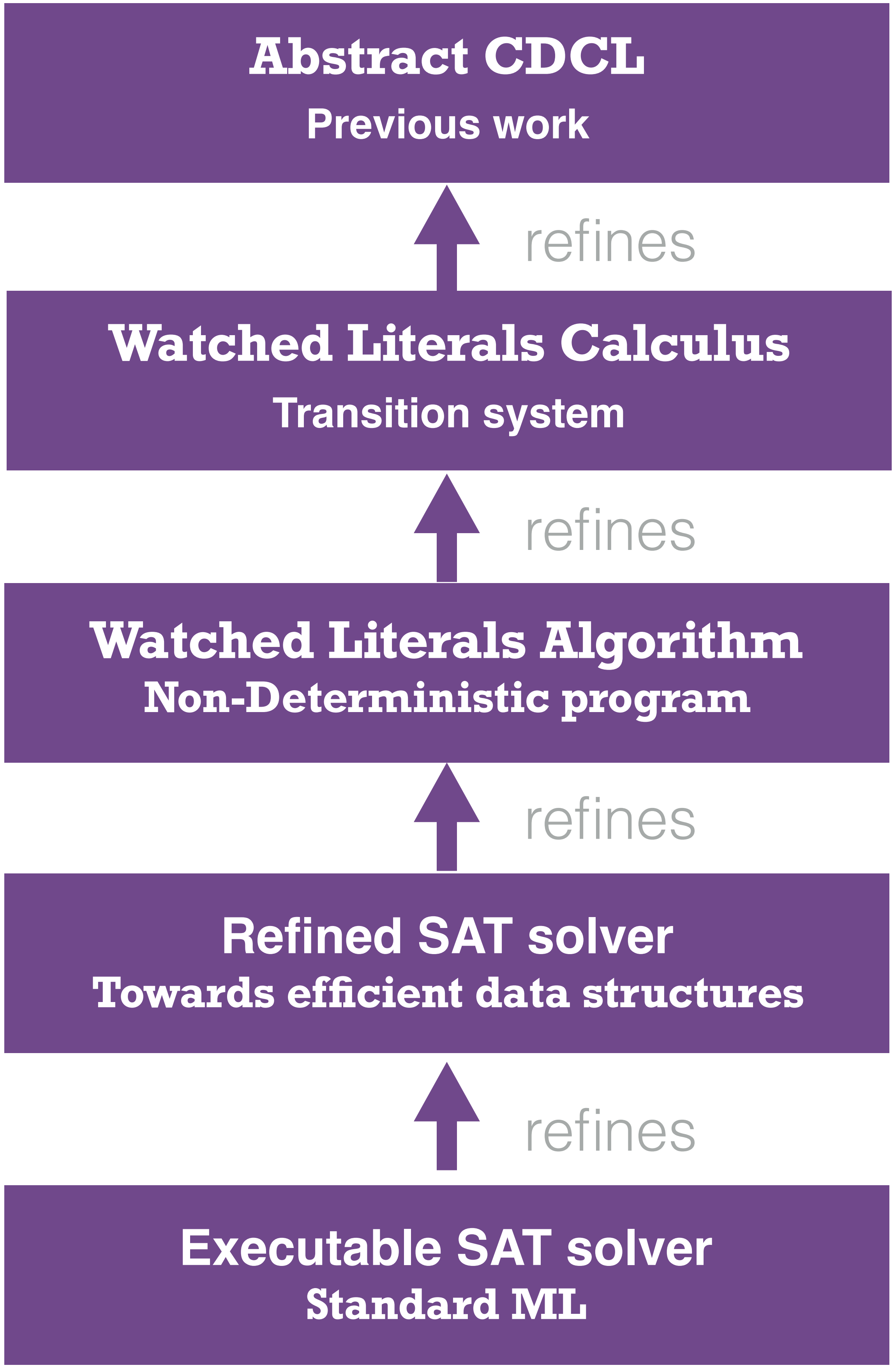
```
C ∨ L ∈ N ⟹ M ⊨as ¬C ⟹ undefined_lit M L ⟹
   (M, N) ⇒CDCL (L # M, N)
```

in Isabelle

## Propagate rule

$$C \lor L \in N \Longrightarrow M \models_{as} \neg C \Longrightarrow \mathtt{undefined\_lit}\ M\ L \Longrightarrow (M, N) \Rightarrow_{\mathrm{CDCL}} (L \mathbin{\#} M, N)$$

Problem:
Iterating over the clauses
is inefficient



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# Watched Literals Calculus

## Transition system

# Watched literals invariant

1. Watch one true literal
2. or watch two unset literals
3. or watch a false literal
   if all other literals are false

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unless a conflict has been found

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unless a conflict has been found

or an update is pending

# Watched literals invariant (less wrong)

1. Watch one true literal
2. or watch two unset literals
3. or watch a false literal
   if all other literals are false

this literal has been set earlier

unless a conflict has been found

or an update is pending

# Watched literals invariant

1. Watch one true literal
2. or watch two unset literals
3. or watch a false literal
   if all other literals are false

→

1. Watch any literal
   if there is a true literal
2. or watch two unset literals
3. or watch a false literal
   if all other literals are false

## Watched literals invariant

1. Watch one true literal
2. or watch two unset literals
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## with blocking literals

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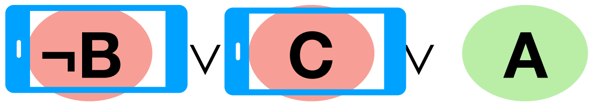

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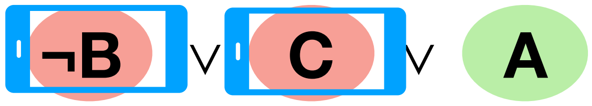

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~~(not yet refined to code)~~

- 🔑 Finding invariants (11 new ones)
- ✗ No high-level description
- ✓ sledgehammer

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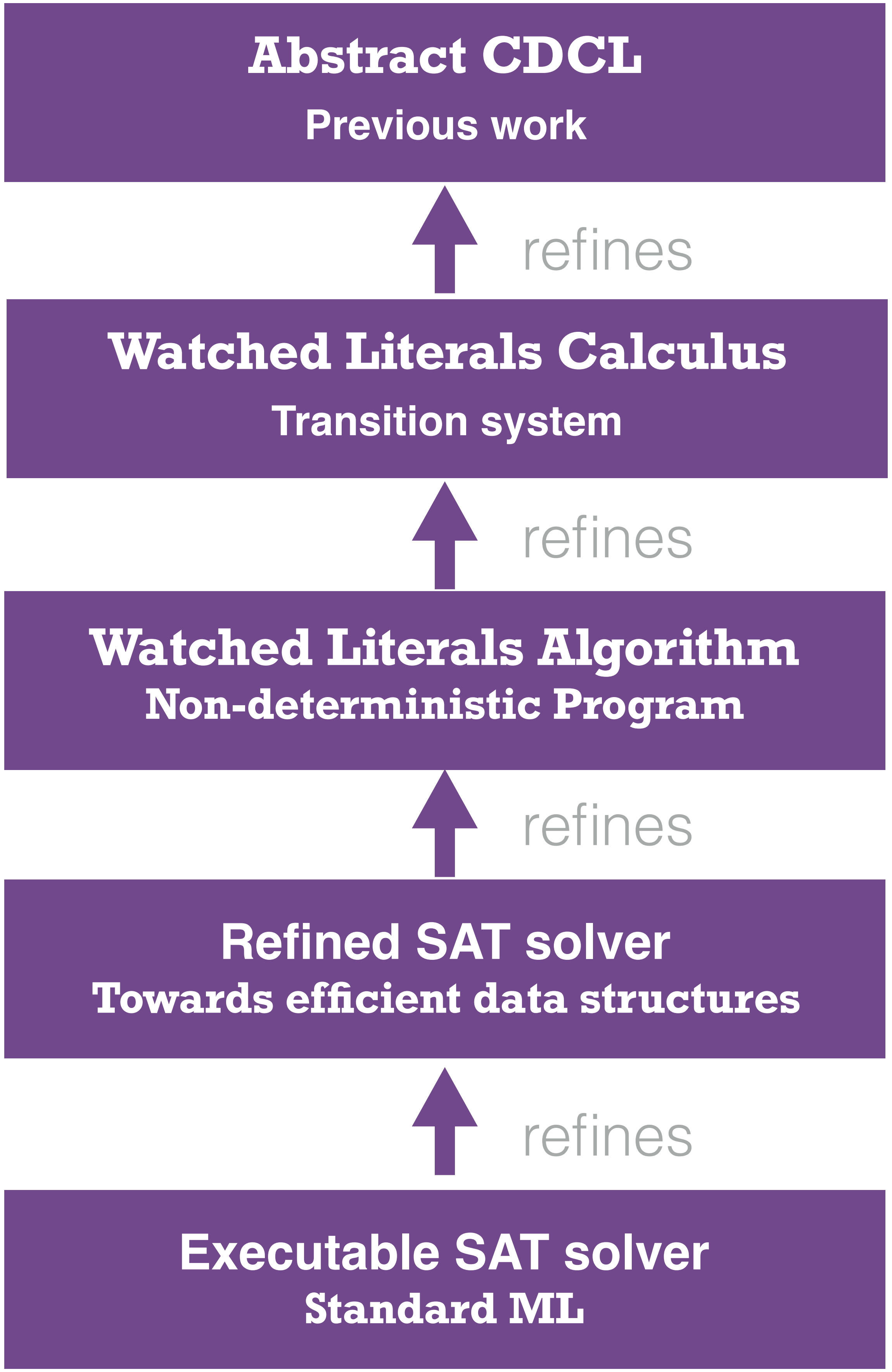
## Correctness theorem

in Isabelle

If S is well-formed and $\mathtt{S} \Rightarrow_{\mathtt{TWL}}{}^{!}\ \mathtt{T}$ then

$$\mathtt{CDCL\_of\ S} \Rightarrow_{\mathtt{CDCL}}{}^{!}\ \mathtt{CDCL\_of\ T}$$



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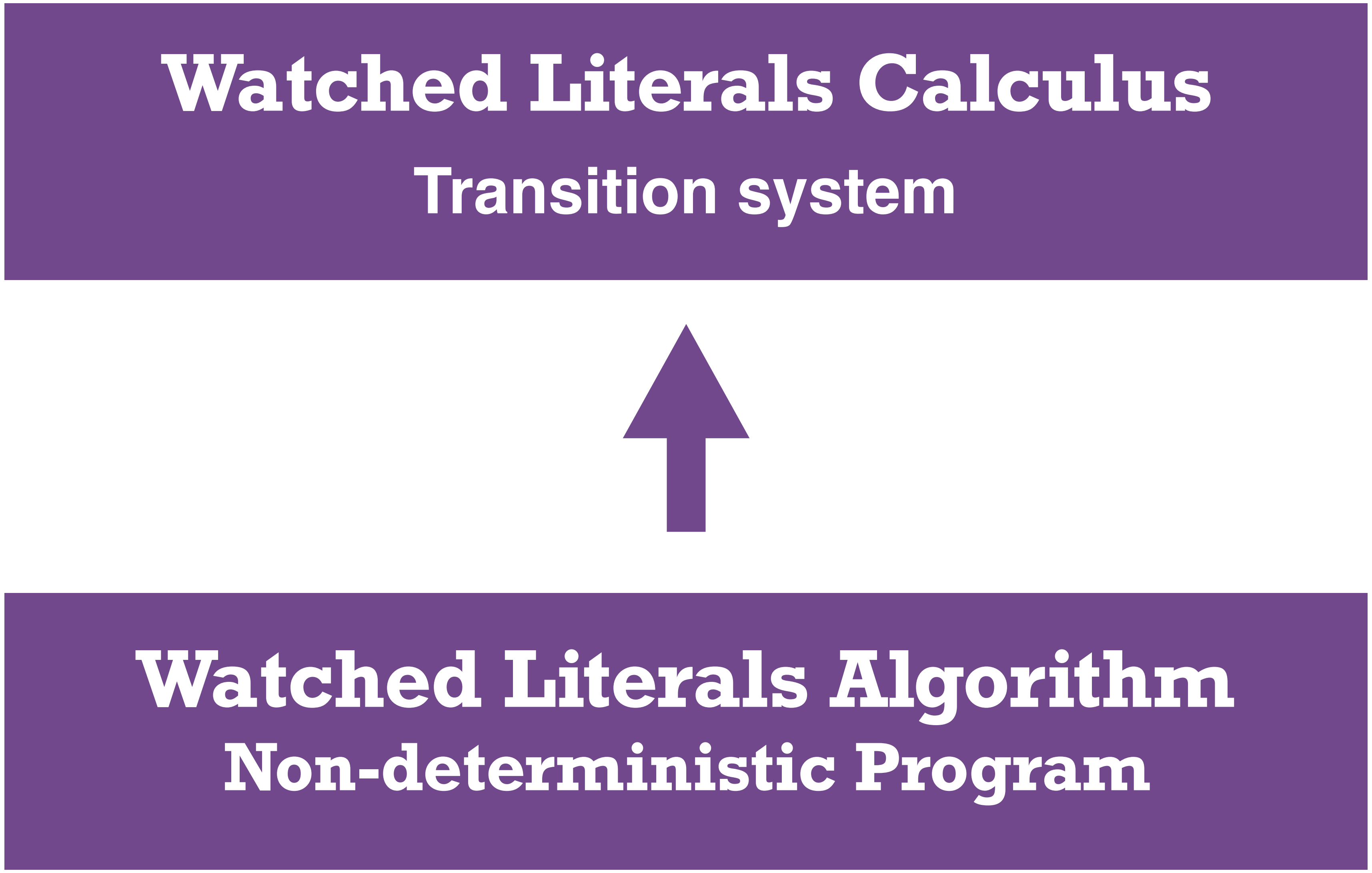

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# DEMO I

# Picking Next Clause

```
propagate_conflict_literal L S :=
  WHILE$_T$
    ($\lambda$T. clauses_to_update T $\neq$ {})

    ($\lambda$T. do {
      ASSERT(clauses_to_update T $\neq$ {})
      C $\leftarrow$ SPEC ($\lambda$C. C $\in$ clauses_to_update T);
      U $\leftarrow$ remove_from_clauses_to_update C T;
      update_clause (L, C) U
     }
    )

    S
```

# Refinement Framework:
# non-deterministic exception monad

```
propagate_conflict_literal L S :=
  WHILE$_T$
    ($\lambda$T. clauses_to_update T $\neq$ {})

    ($\lambda$T. do {
       ASSERT(clauses_to_update T $\neq$ {})
       C ← SPEC ($\lambda$C. C $\in$ clauses_to_update T);
       U ← remove_from_clauses_to_update C T;
       update_clause (L, C) U
     }
    )

    S
```

## Refinement Framework:
## non-deterministic exception monad

**Assertions**

```
propagate_conflict_literal L S :=
  WHILE$_T$
    (λT. clauses_to_update T ≠ {})

    (λT. do {
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      C ← SPEC (λC. C ∊ clauses_to_update T);
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       U ← remove_from_clauses_to_update C T;
       update_clause (L, C) U
     }
    )

    S
```

**Non-deterministic getting of a clause**

## Refinement Framework: non-deterministic exception monad

```
propagate_conflict_literal L S :=
  WHILE_T
    (λT. clauses_to_update T ≠ {})

    (λT. do {
       ASSERT(clauses_to_update T ≠ {})
       C ← SPEC (λC. C ∈ clauses_to_update T);
       U ← remove_from_clauses_to_update C T;
       update_clause (L, C) U
     }
    )

    S
```

- More deterministic (order of the rules)
- But still non deterministic (decisions)
- Goals of the form

- More deterministic (order of the rules)
- But still non deterministic (decisions)
- Goals of the form

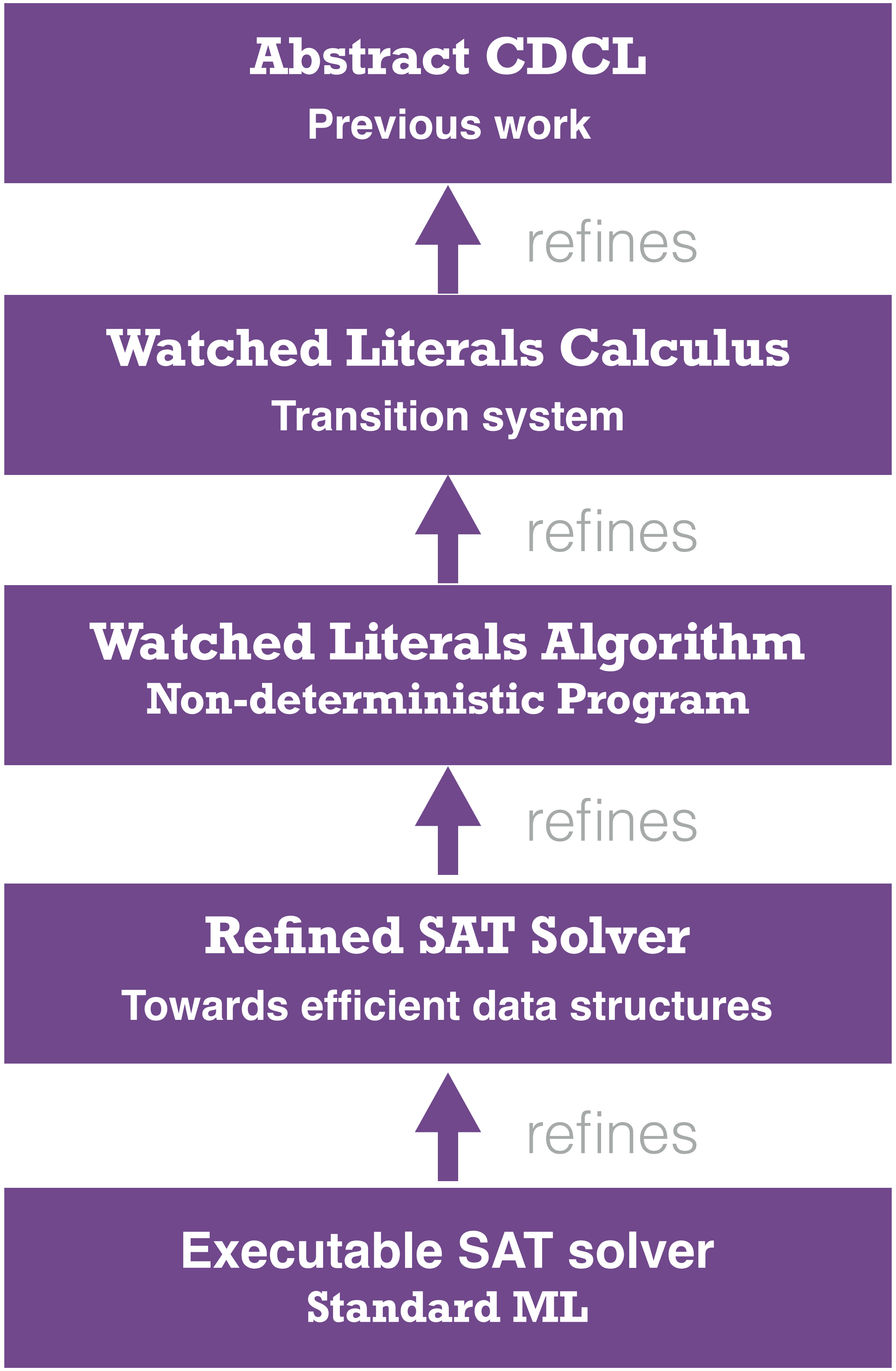
$$\texttt{propagate\_conflict\_literal}\ L\ S \le \texttt{SPEC}(\lambda T.\ S \Rightarrow_{\texttt{TWL}}^{*} T)$$

in Isabelle

✗ VCG’s goals hard to read

✗ Very tempting to write fragile proofs

✓ sledgehammer



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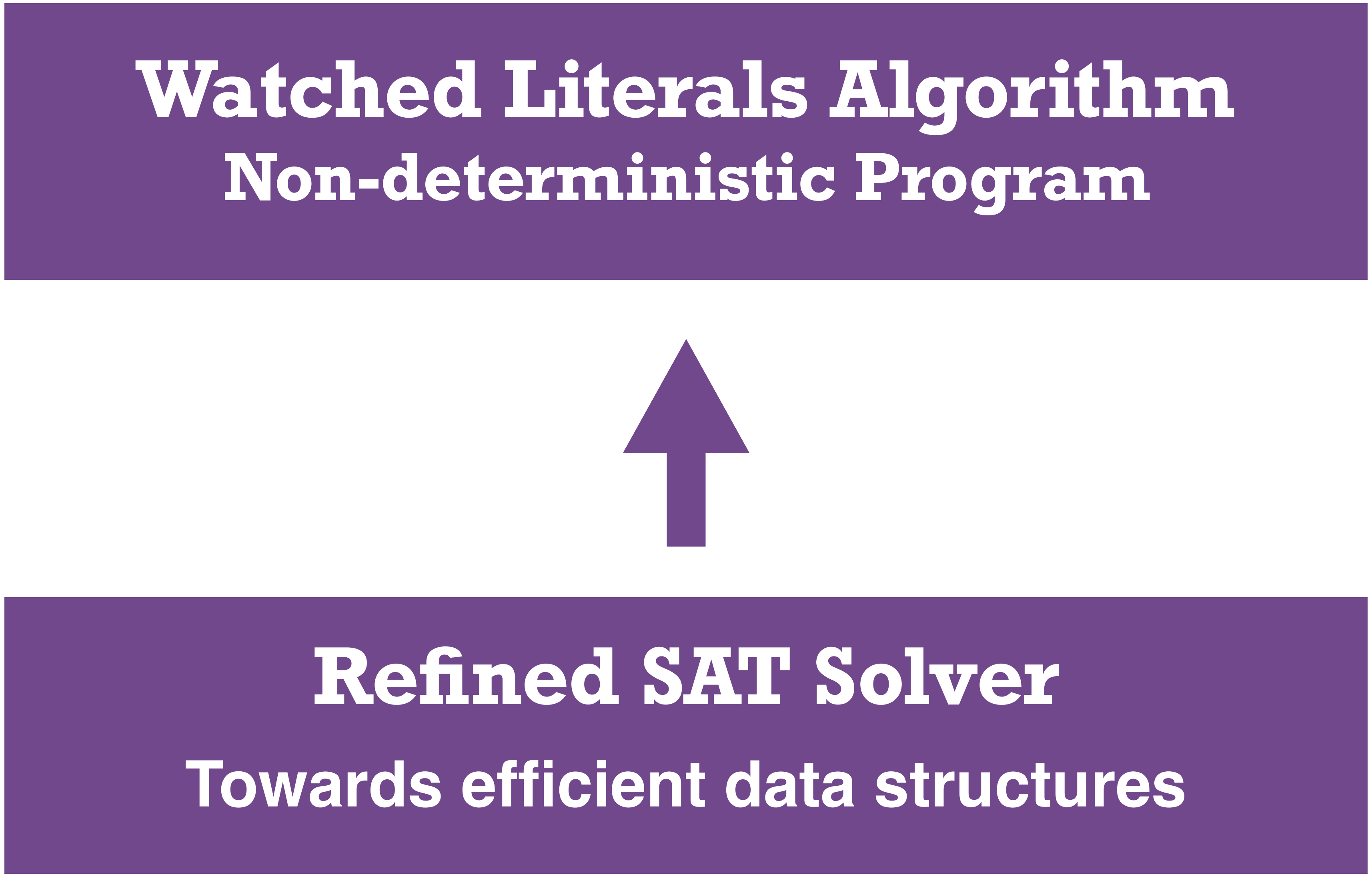


## Watched Literals Algorithm

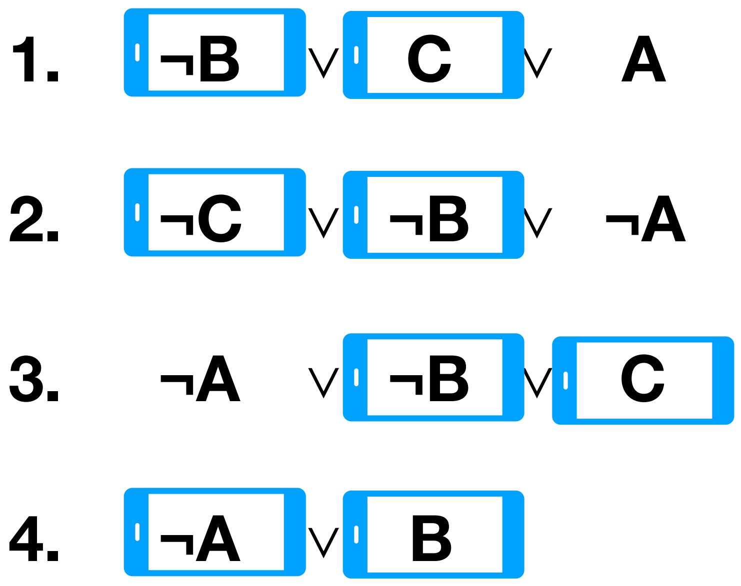
### Non-deterministic Program

## Refined SAT Solver

Towards efficient data structures

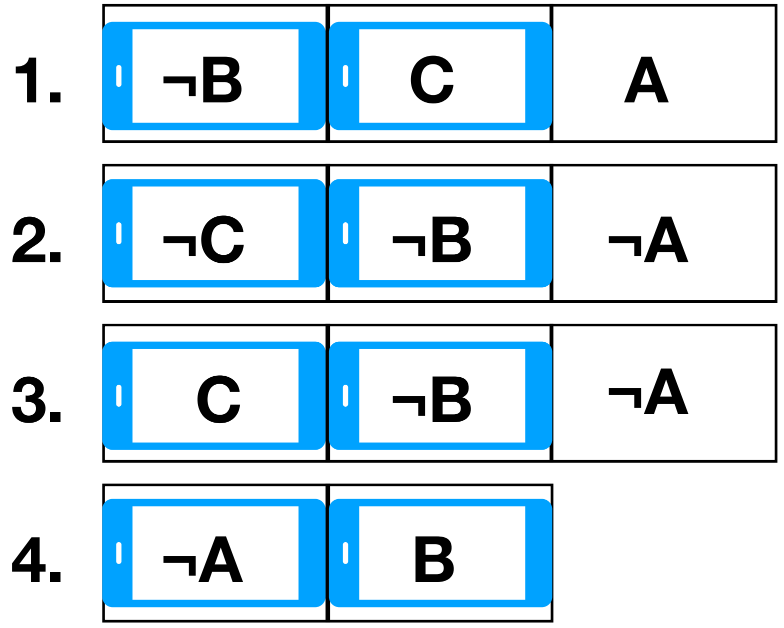
# DPLL with Watched Literals

**Clauses (multisets)**



1. ¬B ∨ C ∨ A
2. ¬C ∨ ¬B ∨ ¬A
3. ¬A ∨ ¬B ∨ C
4. ¬A ∨ B

**Clauses after refinement (lists)**



| | | | |
|---|---|---|---|
| 1. | ¬B | C | A |
| 2. | ¬C | ¬B | ¬A |
| 3. | C | ¬B | ¬A |
| 4. | ¬A | B | |

To update:

A: ¬A: 4 C: 1,3 ¬C: 2

B: 4 ¬B: 1,2,3

- Choice on the heuristics
- Choice on the data structures
- Prepare code synthesis

# Decision heuristic

- Variable-move-to-front heuristic
- No correctness w.r.t. a standard implementation
- Behaves correctly:
  - returns an unset literal if there is one
  - no exception (out-of-bound array accesses)

# DEMO II

```
propagate_conflict_literal L S :=
  WHILE$_T$
    (λT. clauses_to_update T ≠ {})

    (λT. do {
       ASSERT(clauses_to_update T ≠ {})
       C ← SPEC (λC. C ∈ clauses_to_update T);
       U ← remove_from_clauses_to_update C T;
       update_clause L C U
     }
    )

    S
```

```
propagate_conflict_literal_list L S :=
  WHILE$_T$
    (λ(w, T). w < length (watched_by T L))

    (λ(w, T). do {
       C ← (watched_by T L) ! w;
       update_clause_list L C T
     }
    )

    (S, 0)
```

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```
propagate_conflict_literal L S :=
  WHILE_T
    (λT. clauses_to_update T ≠ {})

    (λT. do {
       ASSERT(clauses_to_update T ≠ {})
       C ← SPEC (λC. C ∈ clauses_to_update T);
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       update_clause L C U
     }
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    S
```

```
propagate_conflict_literal_list L S :=
  WHILE_T
    (λ(w, T). w < length (watched_by T L))

    (λ(w, T). do {
       C ← (watched_by T L) ! w;
       update_clause_list L C T
     }
    )

    (S, 0)
```

```
propagate_conflict_literal_list L S ≤ ⇓ conversion_between_states
    (propagate_conflict_literal L T)
```

in Isabelle

In Isabelle

many simp rules along:

$$(S_{\mathrm{list}}, T_{\mathrm{mset}}) \in R_{\mathrm{list\_mset}} \Longrightarrow \mathrm{trail}_{\mathrm{mset}}\ T = \mathrm{trail}_{\mathrm{list}}\ S$$

many invariant along:

$$(\exists T_{\mathrm{mset}}.\ (S_{\mathrm{list}}, T_{\mathrm{mset}}) \in R_{\mathrm{list\_mset}} \wedge \mathrm{inv}_{\mathrm{mset}}\ T) \wedge \mathrm{inv}_{\mathrm{mset}}\ T$$

# Fly, you fool

```
lemma
  ‹P S ⟹ ∃S. P S› for S :: ‹'a × 'b›
  by auto
```

# Fly, you fool

```
lemma
  ‹P S ⟹ ∃S. P S› for S :: ‹'a × 'b›
  by auto
```

## How to deactivate this in Isabelle

```
text ‹Find a less hack-like solution›
setup ‹map_theory_claset
  (fn ctxt => ctxt delSWrapper "split_all_tac")›
```

```
lemma
  fixes S :: ‹'a × 'b × 'c›
  assumes
   H: ‹∃T. (S,T) ∈ R ∧ P (fst S)› and
   [simp]: ‹⋀S T. (S, T) ∈ R ⟹ fst T = fst S›
 shows
   ‹∃T. (S, T) ∈ R ∧ P (fst T)›
  using H
  by auto
```

**vs**

```
lemma
  fixes S :: ‹'a × 'b × 'c›
  assumes
   H: ‹∃T. (S,T) ∈ R ∧ P (fst S)› and
   [simp]: ‹⋀S T. (S, T) ∈ R ⟹ fst S = fst T›
 shows
   ‹∃T. (S, T) ∈ R ∧ P (fst T)›
  using H
  by auto
```

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refines

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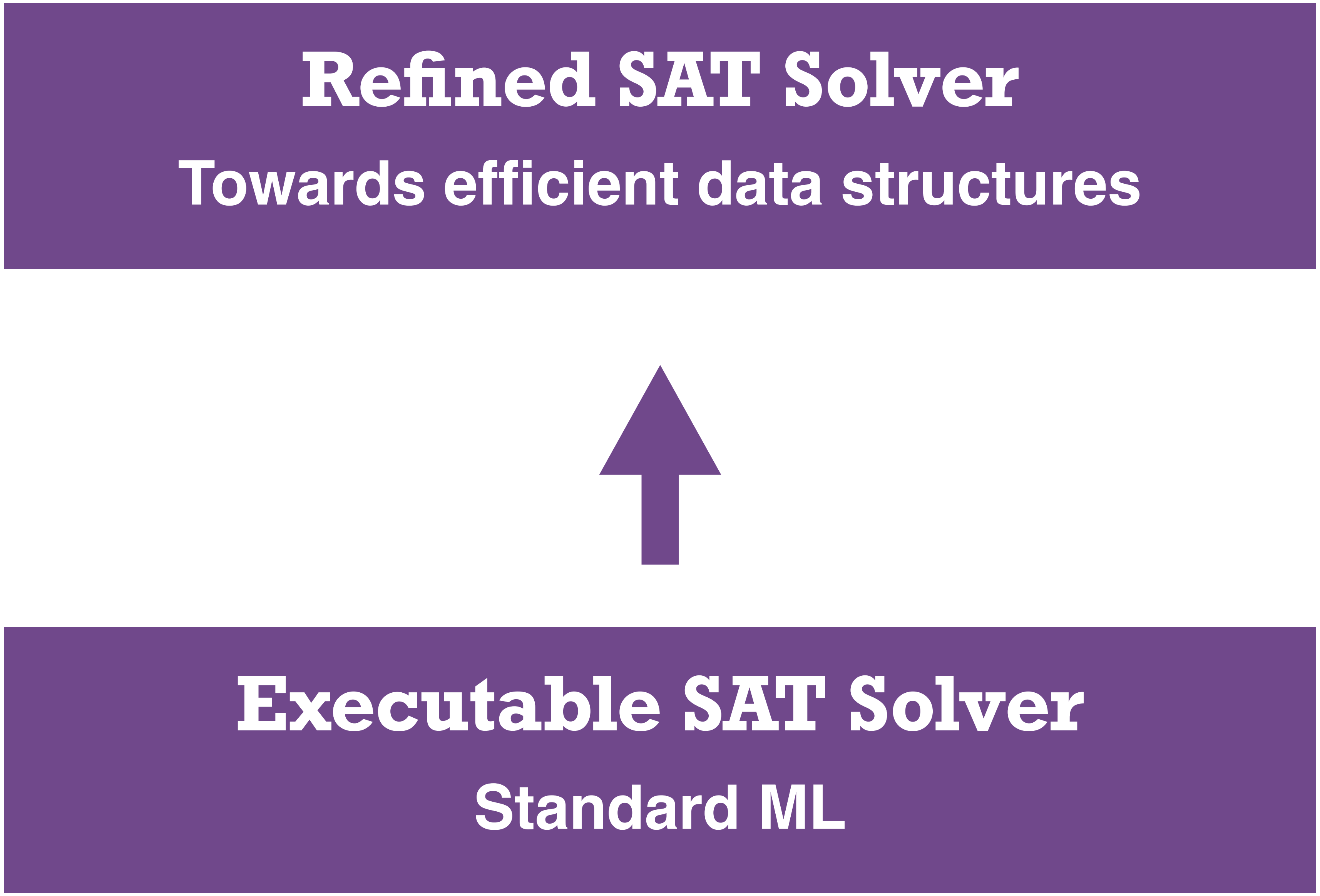

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```
sepref_definition executable_version
  is ‹propagate_conflict_literal_heuristics›
  :: ‹unat_lit_assnᵏ *ₐ state_assnᵈ →ₐ state_assn›
  by sepref
```

Synthesise imperative code and a refinement relation

```
sepref_definition executable_version
  is ‹propagate_conflict_literal_heuristics›
  :: ‹unat_lit_assnᵏ *ₐ state_assnᵈ →ₐ state_assn›
  by sepref
```

Synthesise imperative code and a refinement relation

```
main_loop S :=
   heap_WHILET
       (λ(finished, _). return (¬ finished))
       (λ(_, state).
           propagate state ≫=
           analyse_or_decide)
       (False, state) ≫=
    (λ(_, final_state). return final_state)
```

```
sepref_definition executable_version
  is ‹propagate_conflict_literal_heuristics›
  :: ‹unat_lit_assnᵏ *ₐ state_assnᵈ →ₐ state_assn›
  by sepref
```

Synthesise imperative code and a refinement relation

```
fun main_loop state =
 fn () =>
  let
   val (_, final_state) =
    heap_WHILET
      (fn (done, _) => (fn () => not done))
      (fn (_, state) =>
        (analyse_or_decide (propagate state ()) ()))
      (false, xi)
      ();
  in final_state end;
```

```
sepref_definition executable_version
  is ‹propagate_conflict_literal_heuristics›
  :: ‹unat_lit_assn^k *_a state_assn^d →_a state_assn›
  by sepref
```

Synthesise imperative code and a refinement relation

```
fun cdcl_twl_stgy_prog_wl_D_code x =
  (fn xi => fn () =>
    let
      val a =
        heap_WHILET (fn (a1, _) => (fn () => (not a1)))
          (fn (_, a2) =>
            (fn f_ => fn () => f_ ((unit_propagation_outer_loop_wl_D a2) ()) ())
              cdcl_twl_o_prog_wl_D_code)
          (false, xi) ();
    in
      let
        val (_, aa) = a;
      in
        (fn () => aa)
      end
        ()
```

```
in final_state end;
```

## 🔑 Choice on the data structures

Clauses: resizable arrays of (fixed sized) arrays

However, no aliasing

- Indices instead of pointers
- N[C] makes a copy, so only use N[C][i]

✓ Generates imperative code

✗ No error messages

✗ Transformations before generating code

Clauses of length 0 and 1

Once combined with an initialisation:

in Isabelle

```
‹(IsaSAT_code, model_if_satisfiable)
    ∈ [λN. each_clause_is_distinct N ∧
        literals_fit_in_32_bit_integer N]ₐ
      clauses_as_listsᵏ → model›
```

Exported code tested with an unchecked parser
(easy and medium problems from the SAT competition 2009)

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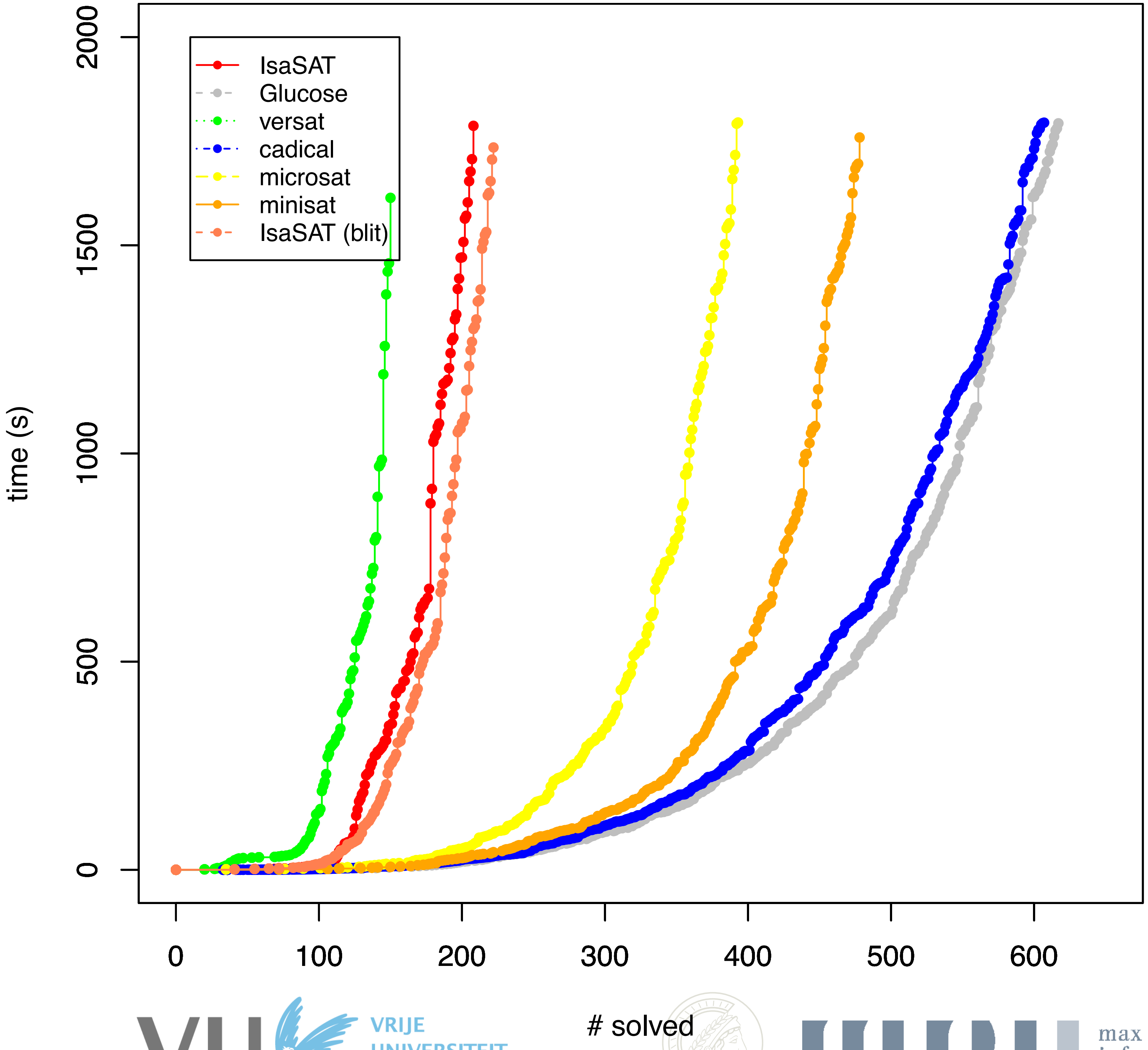
# SAT-Comp ’09, ’15 (main track), and ’14 (all submitted problems), already preprocessed



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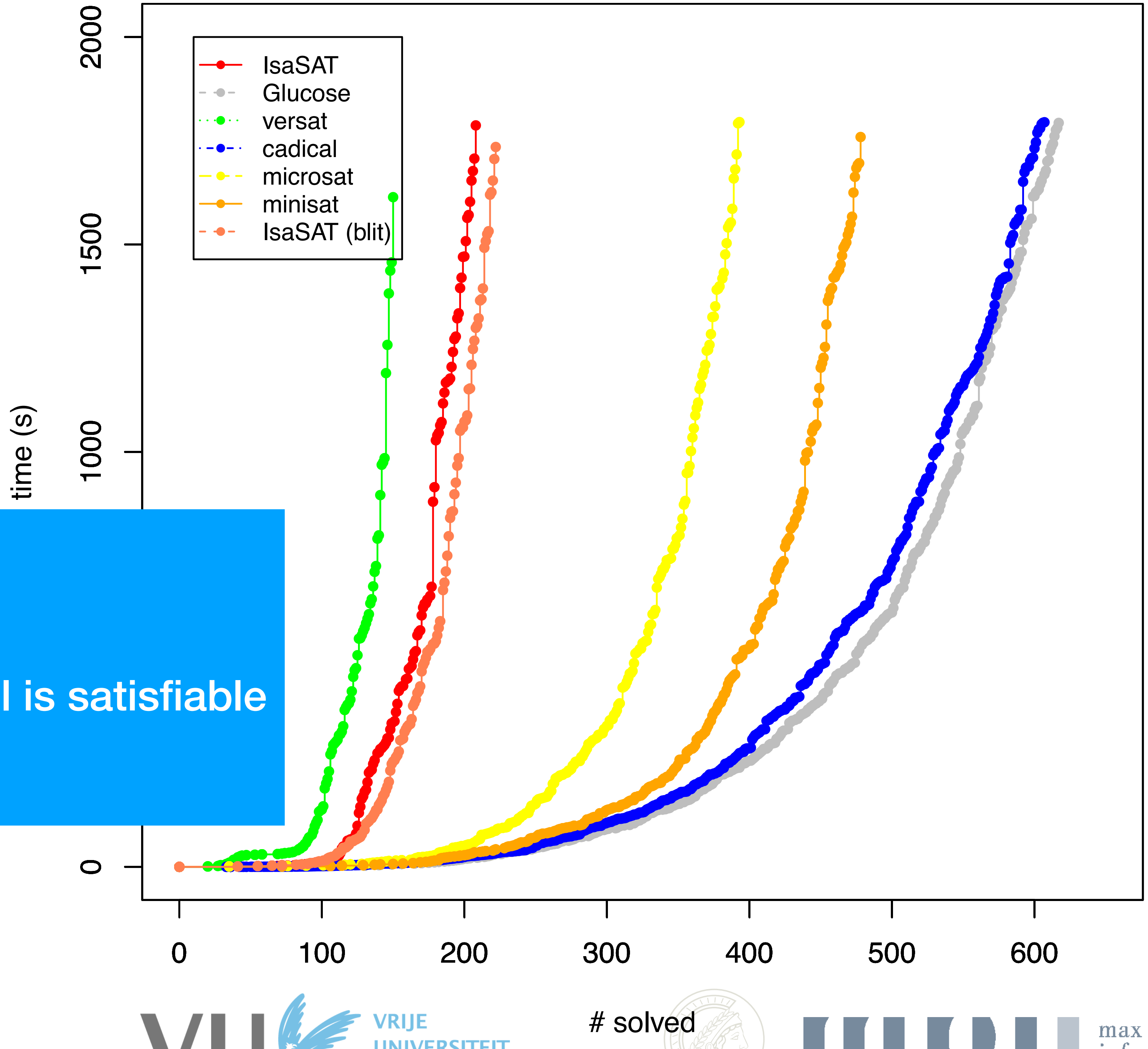
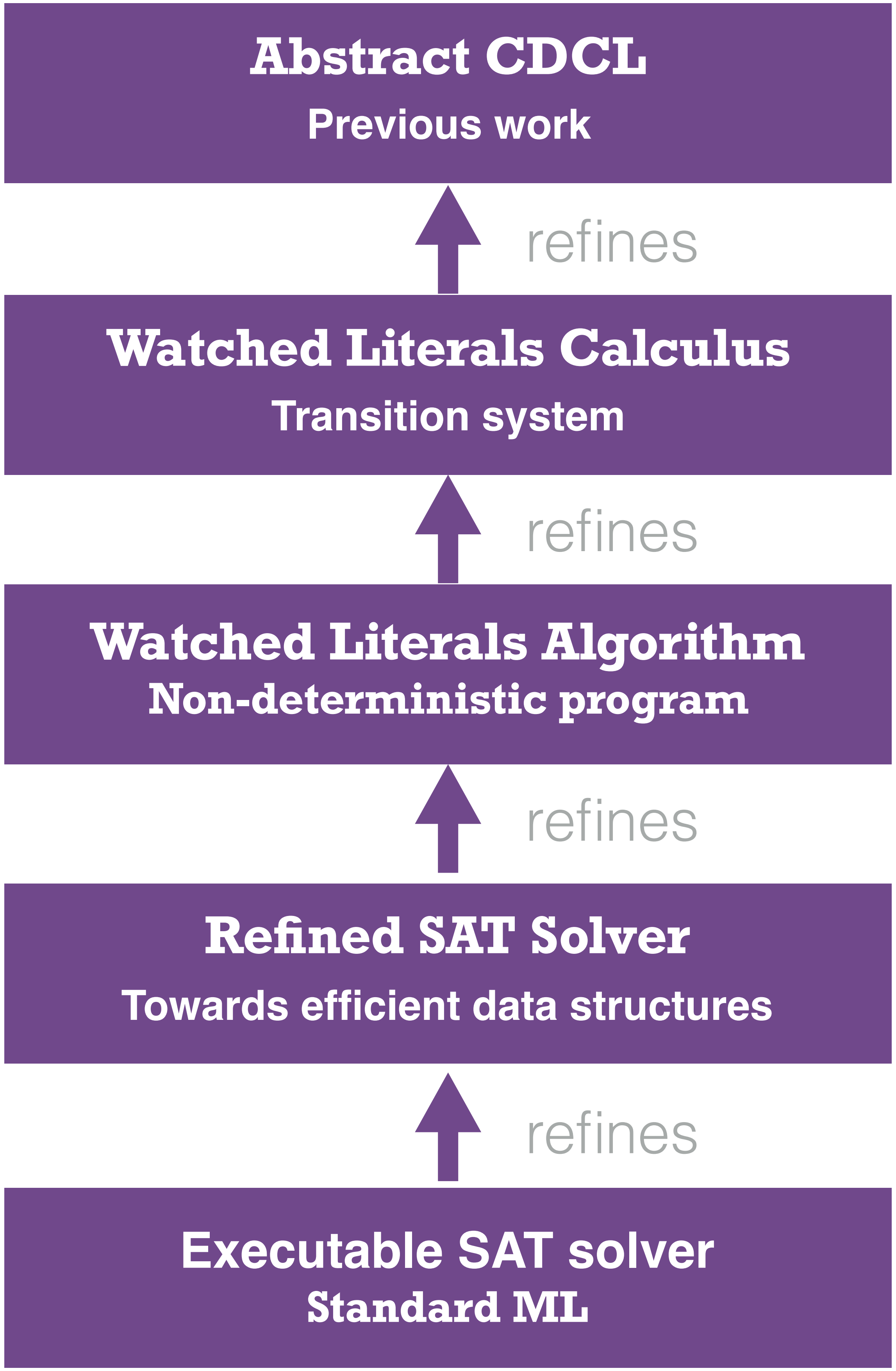


Correct up to:

- run-time checks
- checking the model is satisfiable



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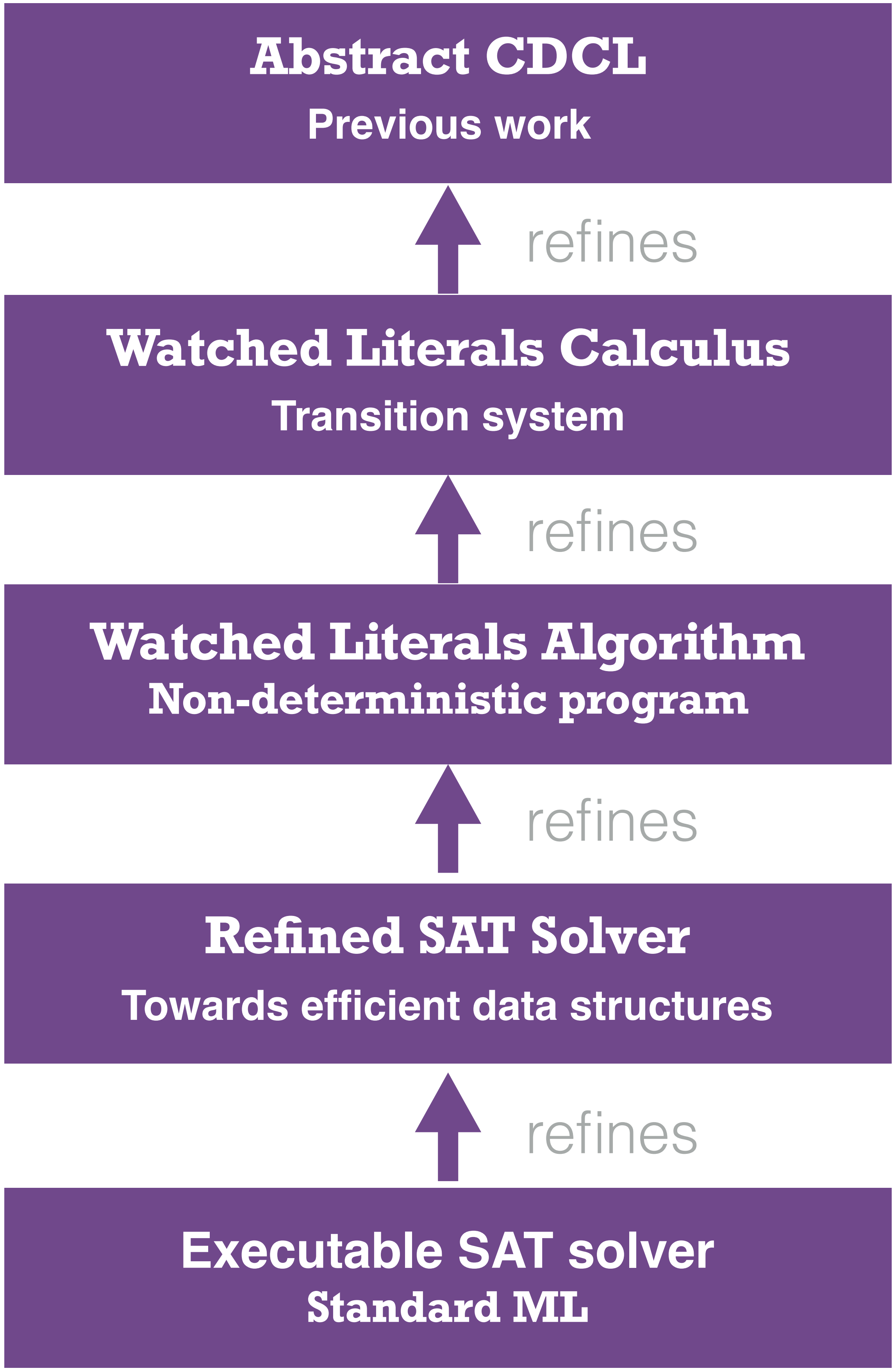

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- better implementation (trail, conflict)
- dynamic decision heuristic

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**Abstract CDCL**

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- allow learned clause minimisation
- no reuse of restarts

refines

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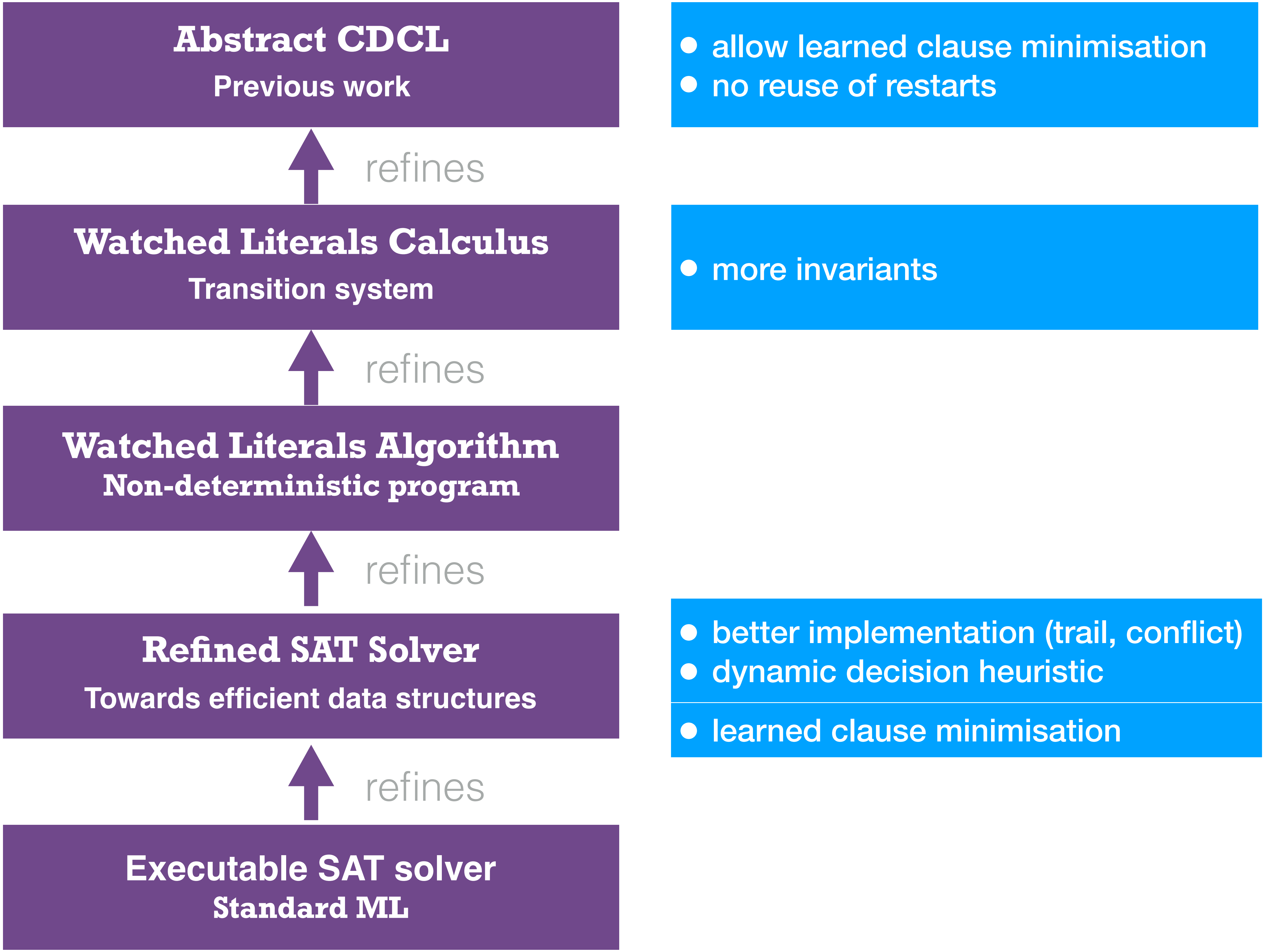
- better implementation (trail, conflict)
- dynamic decision heuristic

- learned clause minimisation

refines

**Executable SAT solver**

**Standard ML**



**Abstract CDCL**
Previous work

- allow learned clause minimisation
- no reuse of restarts

refines

**Watched Literals Calculus**
Transition system

- more invariants

refines

**Watched Literals Algorithm**
**Non-deterministic program**

refines

**Refined SAT Solver**
Towards efficient data structures

- better implementation (trail, conflict)
- dynamic decision heuristic
- learned clause minimisation

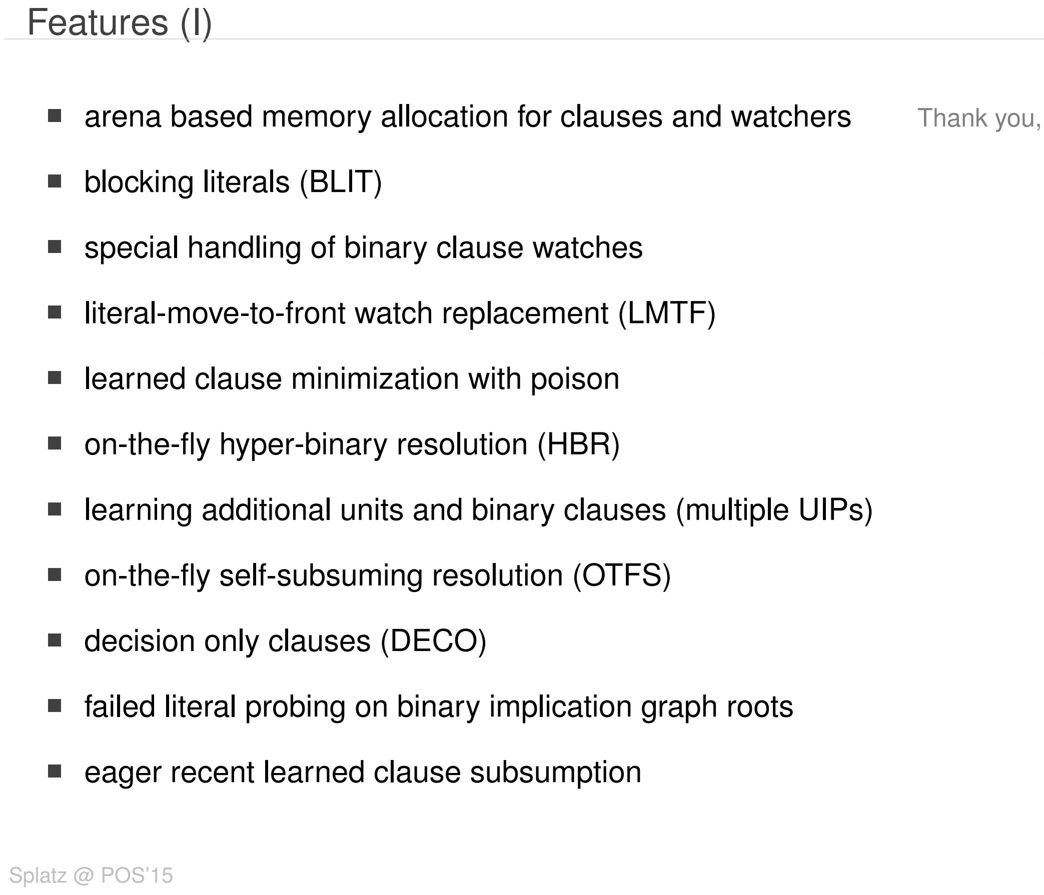
refines

**Executable SAT solver**
**Standard ML**

# How hard is it?

| | Paper | Proof assistant |
|---|---|---|
| **Very abstract** | 13 pages | 50 pages |
| **Abstract CDCL** | 9 pages<br>(½ month) | 90 pages<br>(5 months) |
| **IsaSAT** | 1 page<br>(C++ code of MiniSat) | 900 pages<br>(2 years) |

# How much is missing?



## Features (I)

Thank you, Norbert & Mate!

- arena based memory allocation for clauses and watchers
- blocking literals (BLIT)
- special handling of binary clause watches
- literal-move-to-front watch replacement (LMTF)
- learned clause minimization with poison
- on-the-fly hyper-binary resolution (HBR)
- learning additional units and binary clauses (multiple UIPs)
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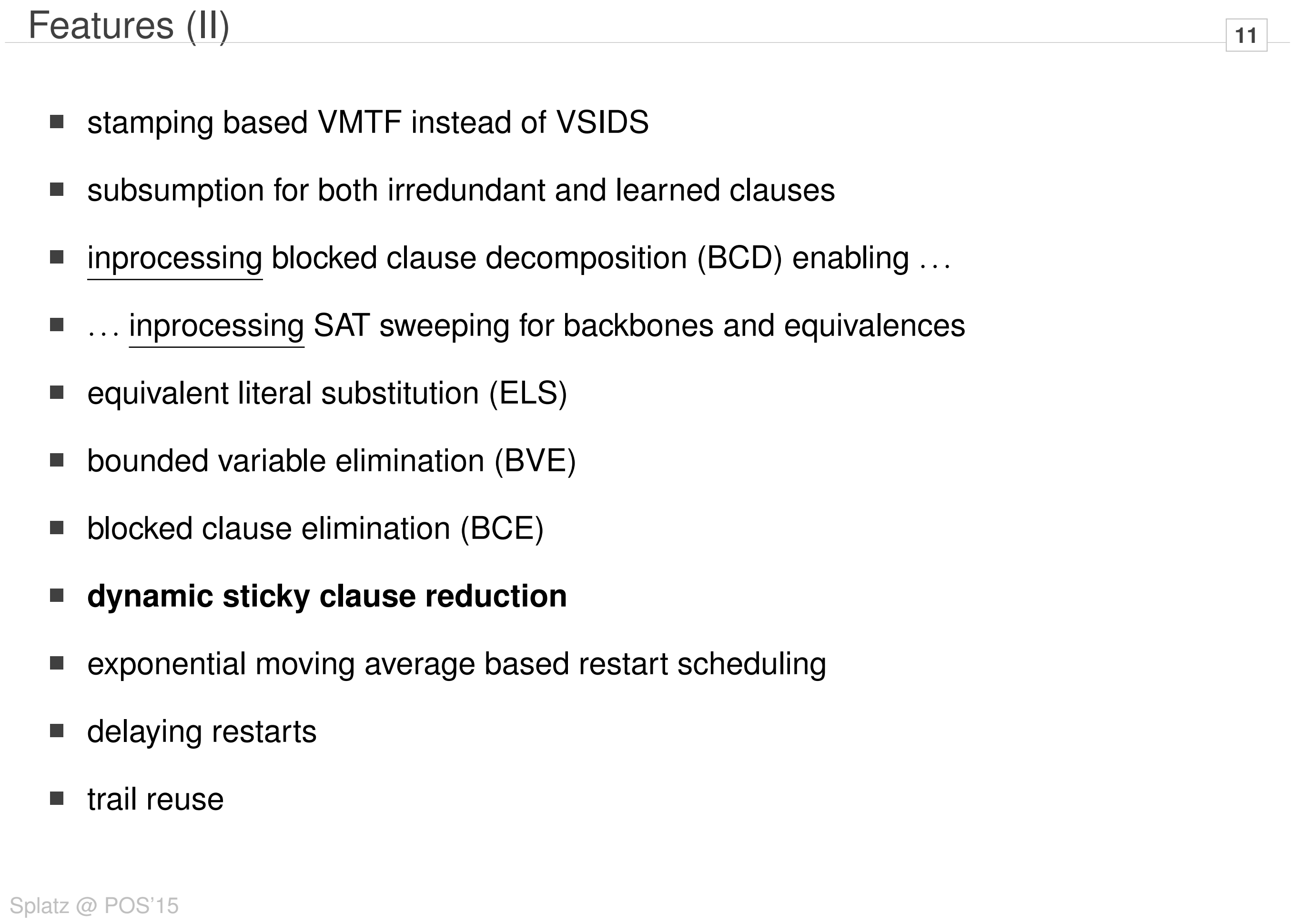
- failed literal probing on binary implication graph roots
- eager recent learned clause subsumption

**Slides by Armin Biere**



## Features (II)

- stamping based VMTF instead of VSIDS
- subsumption for both irredundant and learned clauses
- inprocessing blocked clause decomposition (BCD) enabling . . .
- . . . inprocessing SAT sweeping for backbones and equivalences
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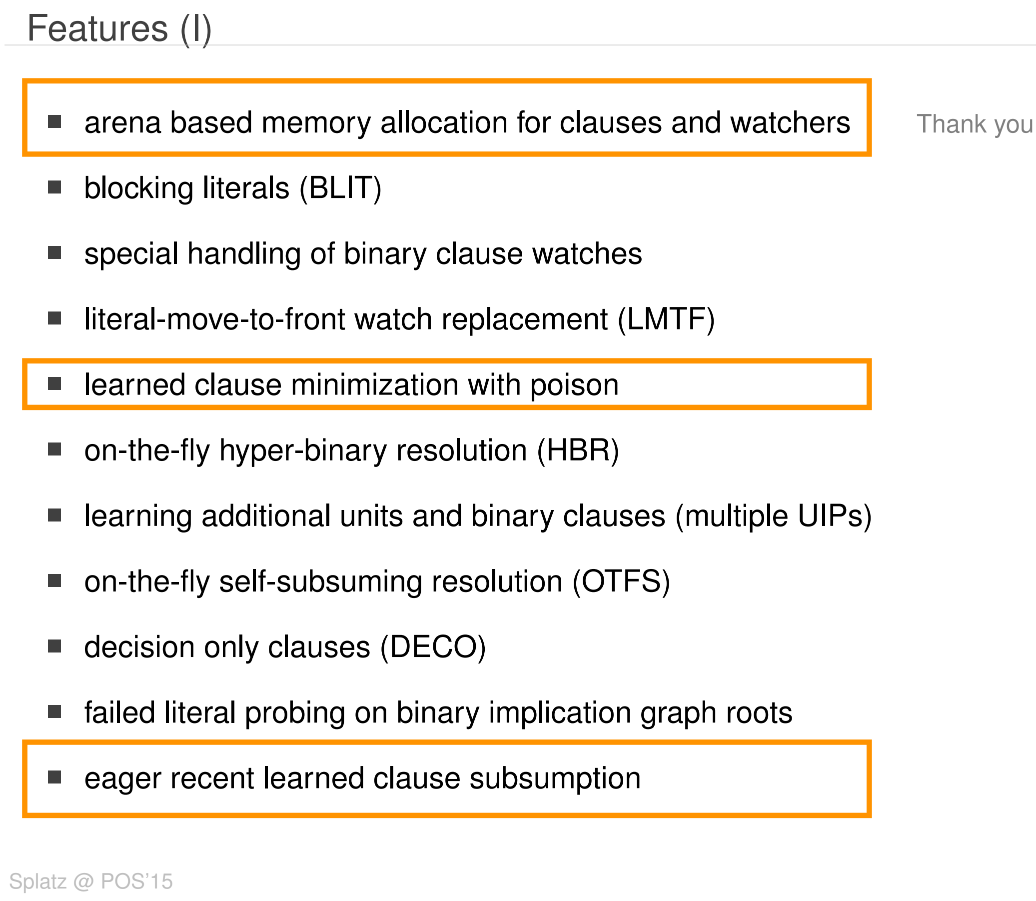

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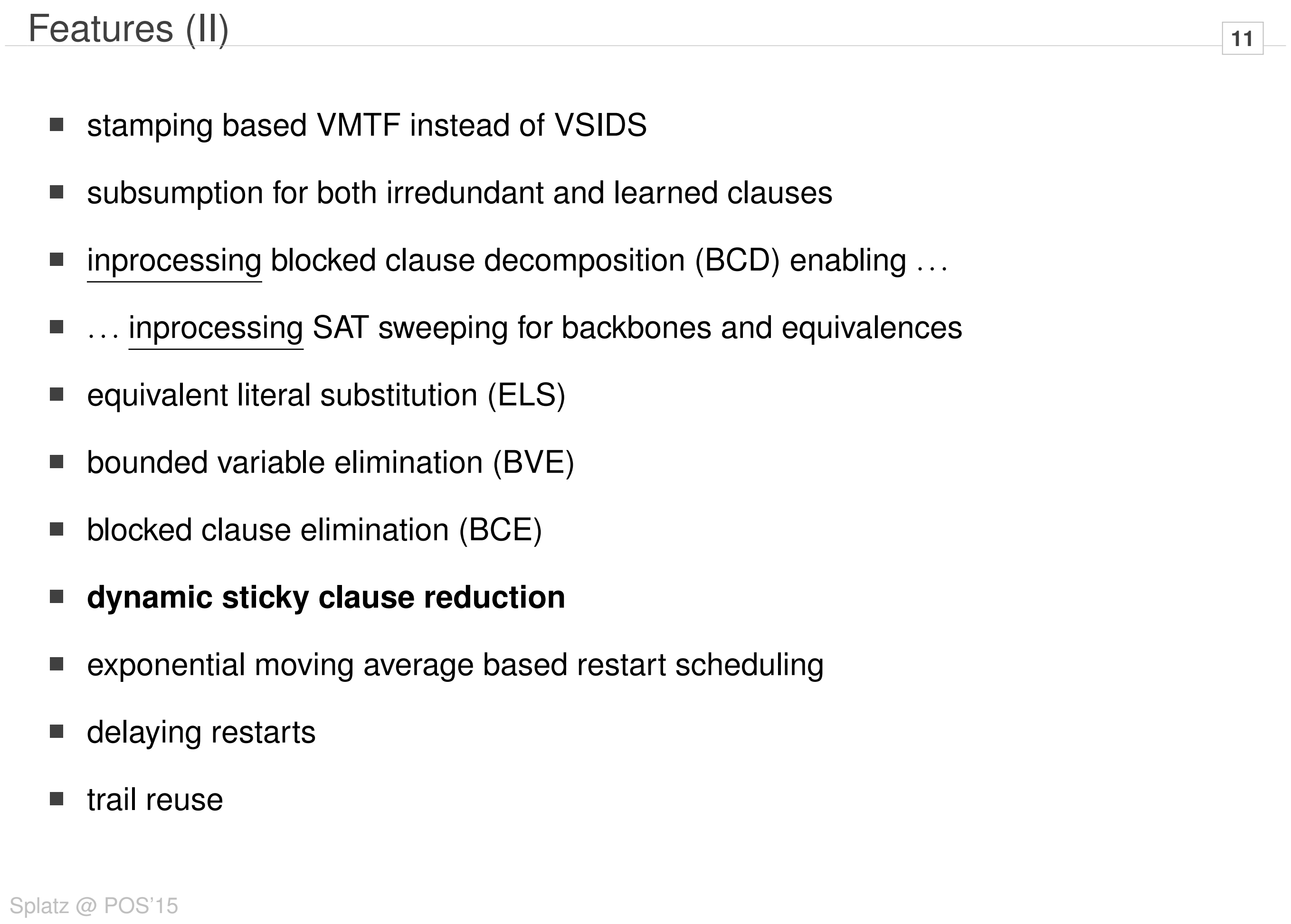
*Code only*

Thank you, Norbert & Mate!

**Slides by Armin Biere**



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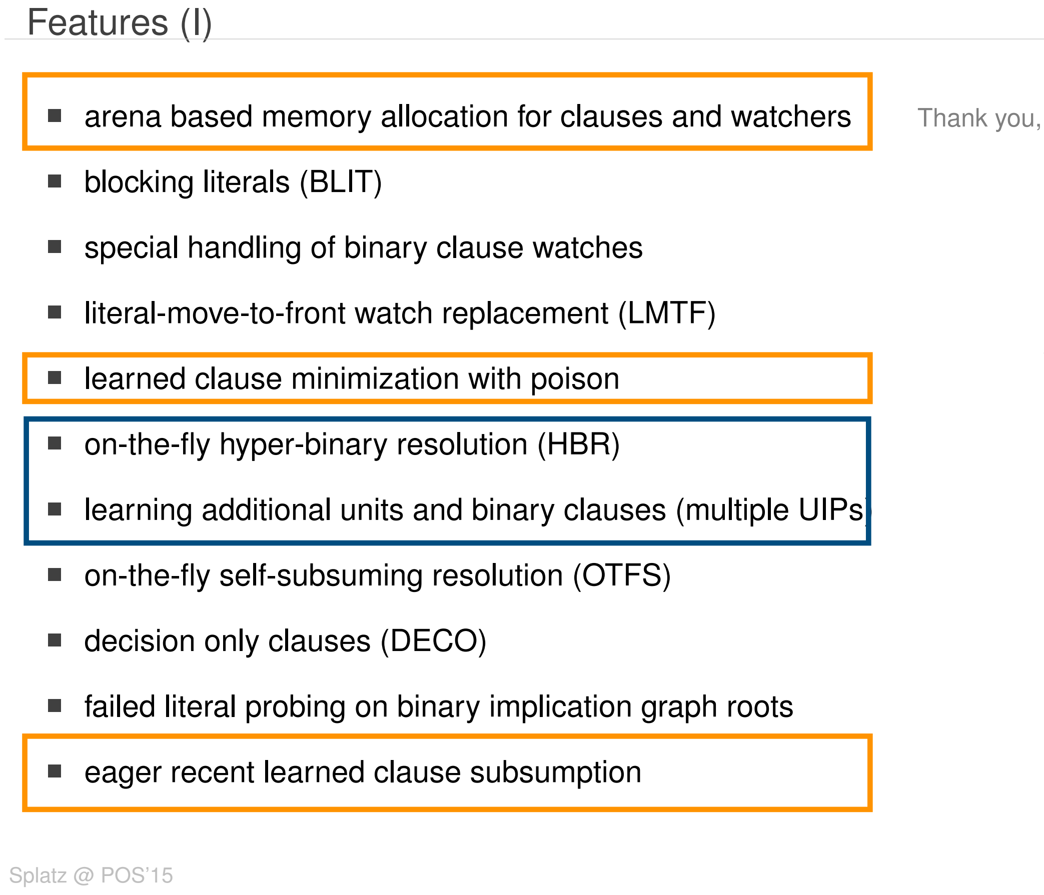

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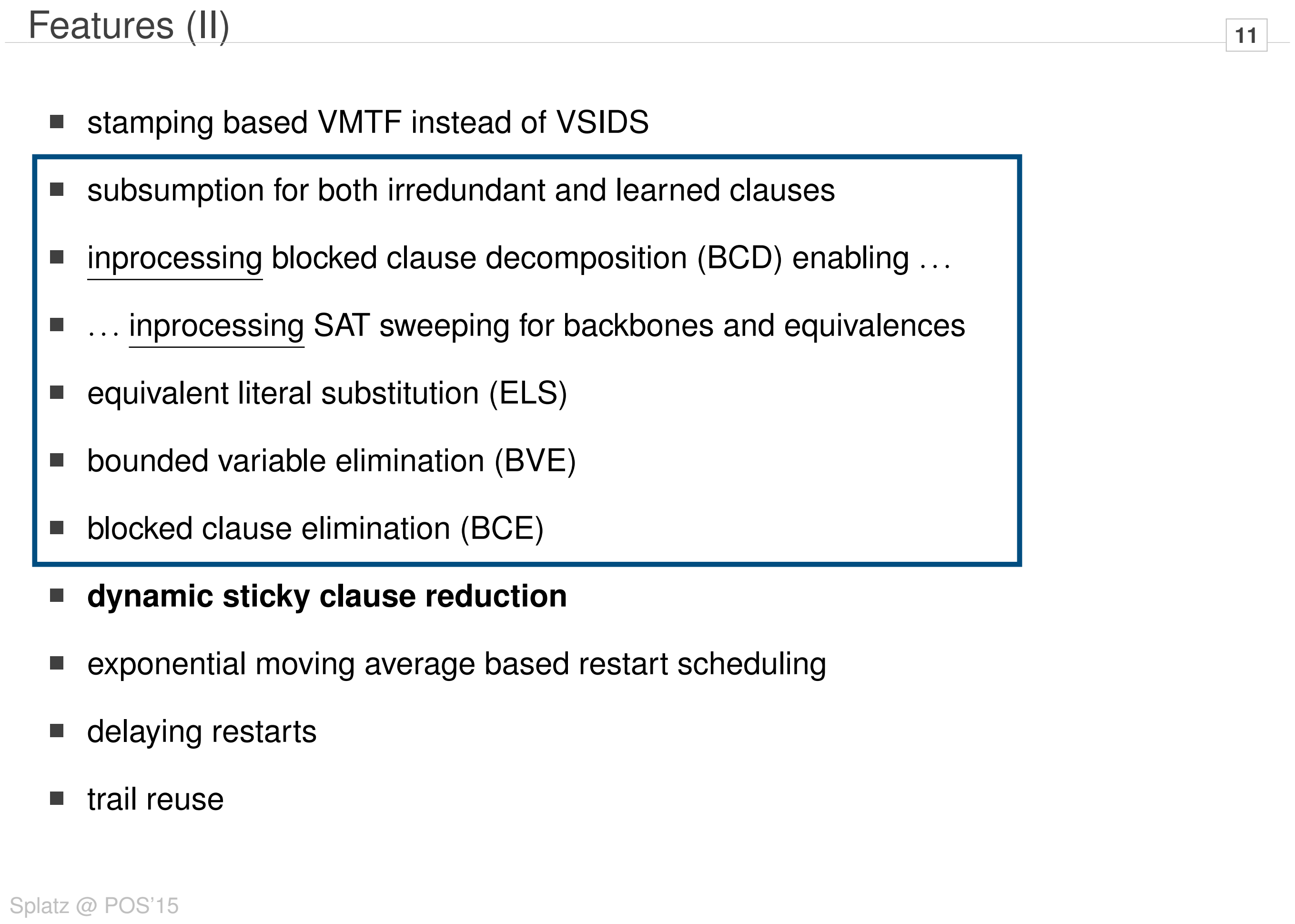

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Splatz @ POS'15

*Code only*

*Strengthening*

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Splatz @ POS'15

**Slides by Armin Biere**

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Splatz @ POS'15

*Code only*

*Strengthening*

*Change CDCL*

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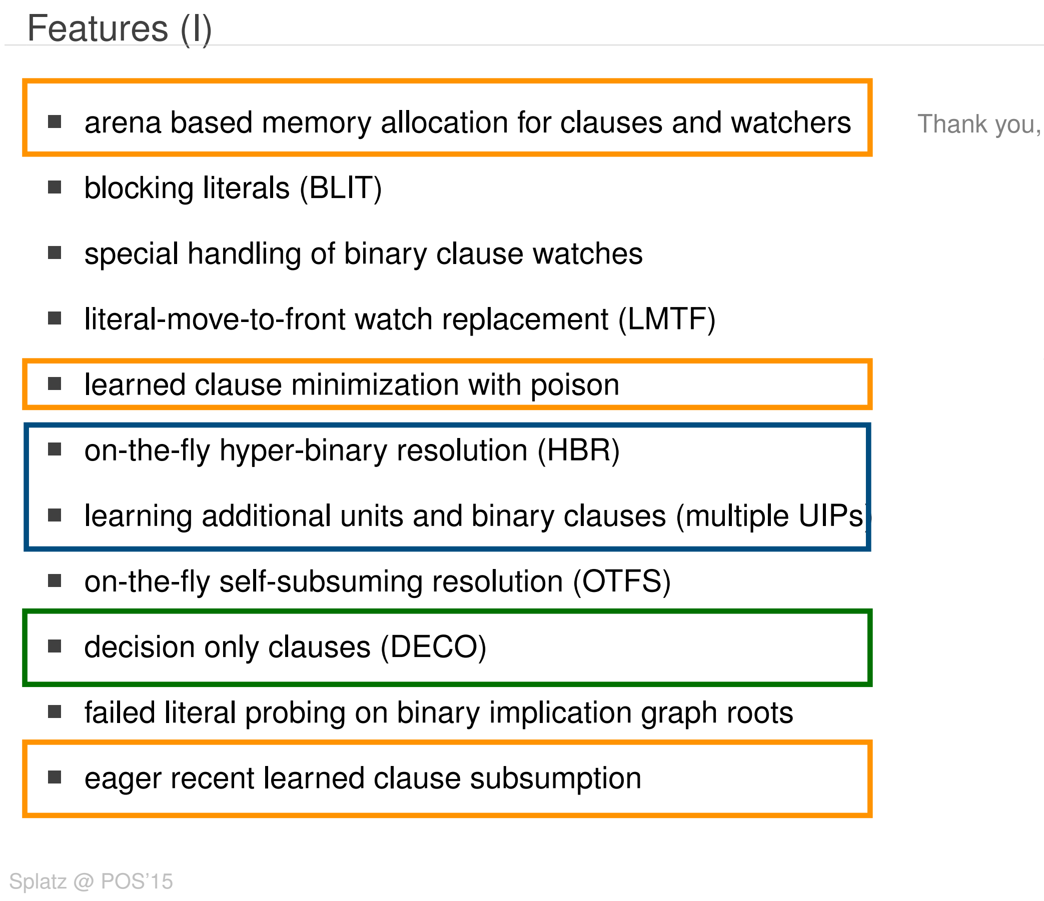

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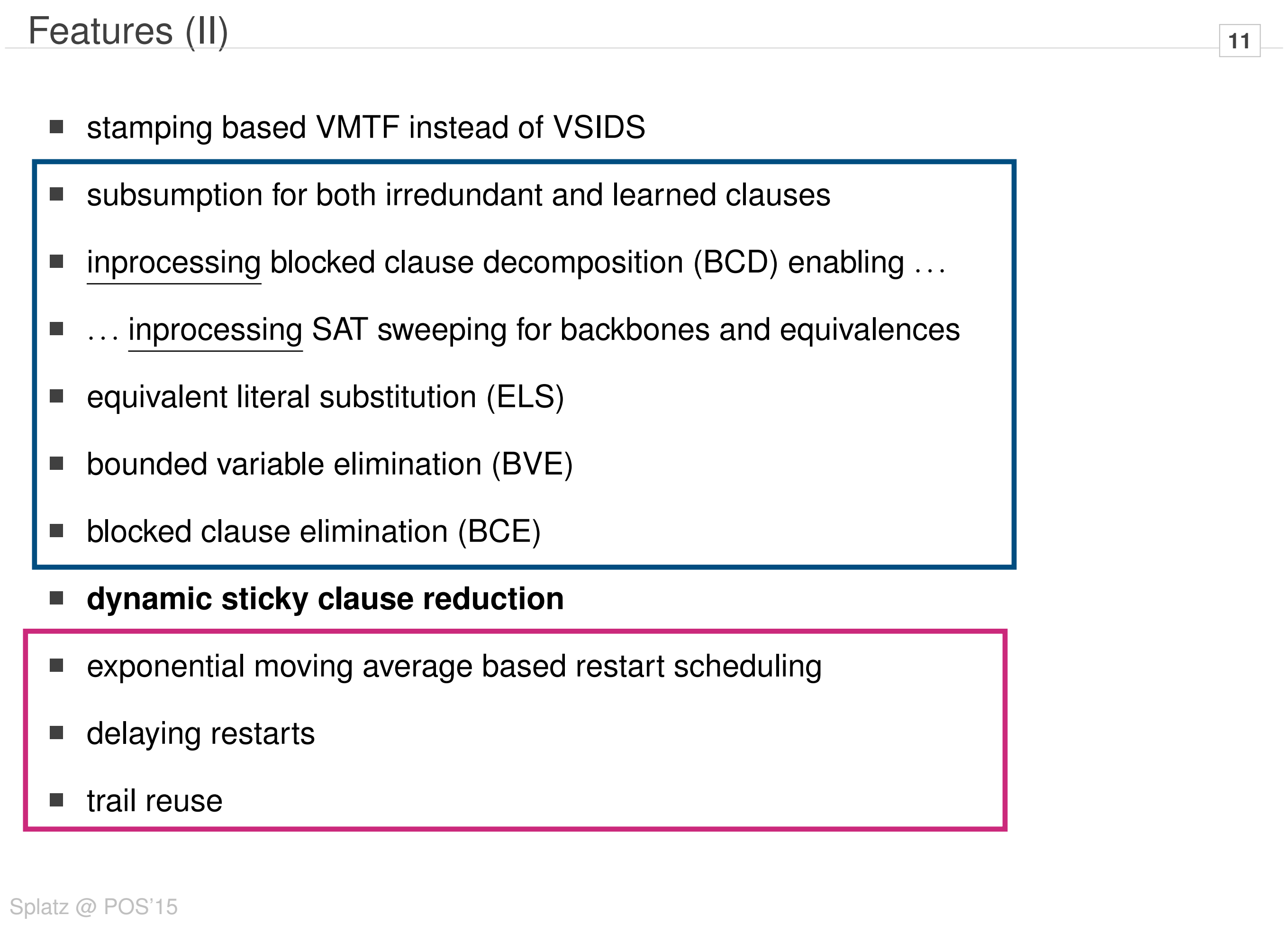

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Splatz @ POS'15

*Code only*

*Restarts (future)*

*Strengthening*

*Change CDCL*

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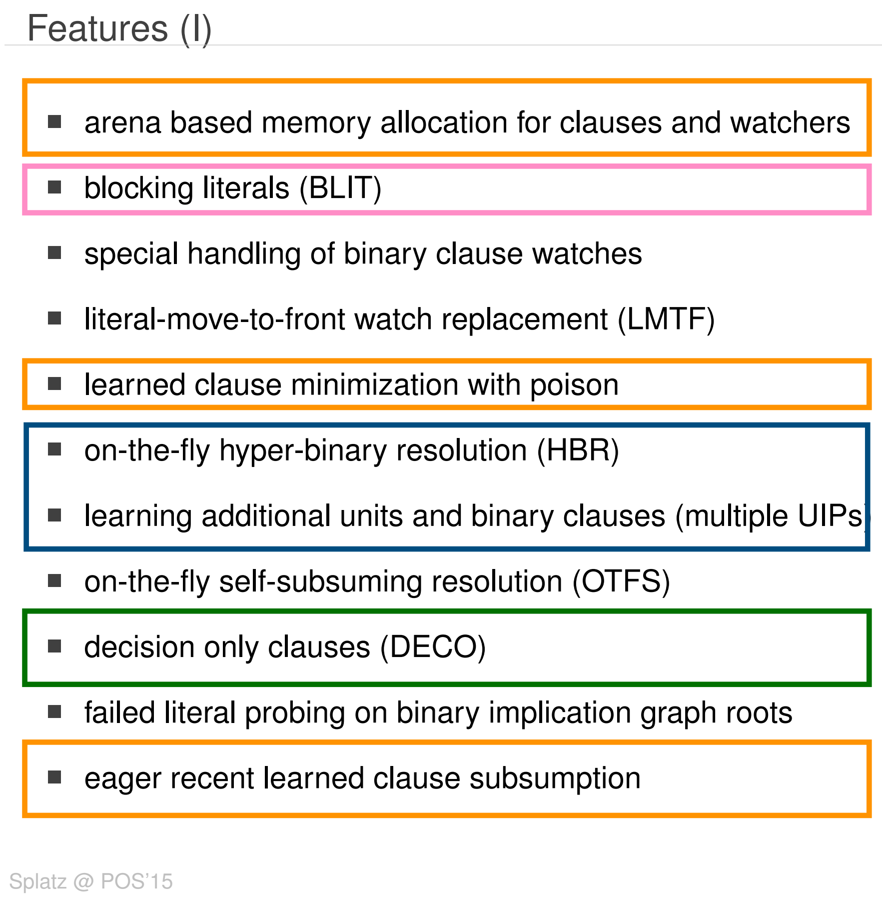

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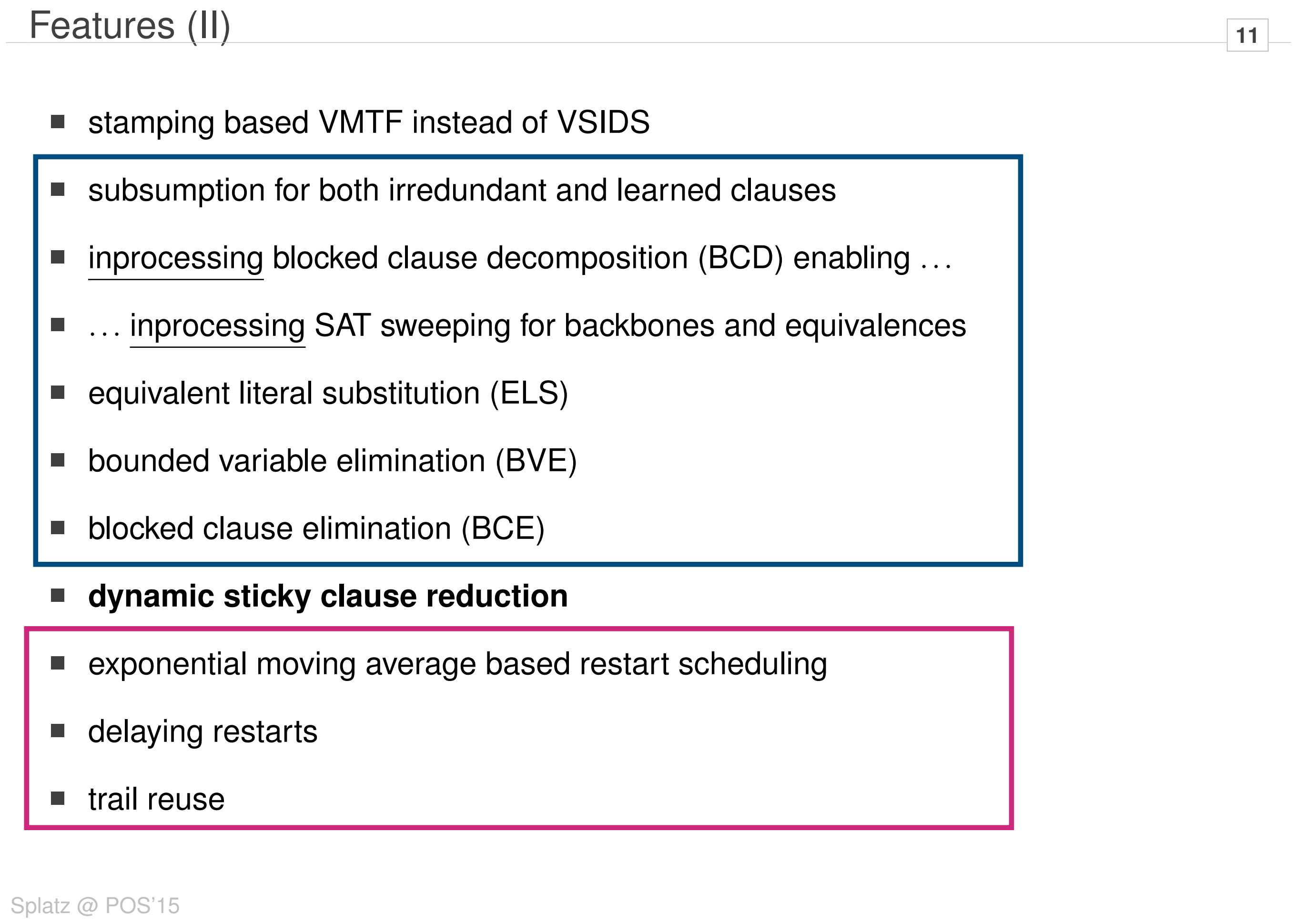

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Splatz @ POS'15

*Code only*

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*Strengthening*

*Change WL*

*Change CDCL*

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Thank you, Norbert & Mate!

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**Slides by Armin Biere**

Features (II)

11

- Unchecked array accesses (Isabelle takes care of it)
- No unbounded integers (in theory, not complete anymore)
- Restarts

- **dynamic sticky clause reduction**
- exponential moving average based restart scheduling
- delaying restarts
- trail reuse

Splatz @ POS'15

*Code only*

*Restarts (future)*

*Strengthening*

*Change WL*

*Change CDCL*

# What is under the carpet? (I)

```
code_printing constant nth_u_code' ⇀ (SML) "(fn/ ()/ =>/ Array.sub/ ((_),/ Word32.toInt (_)))"

code_printing constant nth_u64_code' ⇀ (SML) "(fn/ ()/ =>/ Array.sub/ ((_),/ Uint64.toFixedInt (_)))"

code_printing constant heap_array_set'_u' ⇀
  (SML) "(fn/ ()/ =>/ Array.update/ ((_),/ (Word32.toInt (_)),/ (_)))"

code_printing constant heap_array_set'_u64' ⇀
  (SML) "(fn/ ()/ =>/ Array.update/ ((_),/ (Word64.toInt (_)),/ (_)))"

code_printing constant two_uint32 ⇀ (SML) "(Word32.fromInt 2)"

code_printing constant length_u_code' ⇀ (SML_imp) "(fn/ ()/ =>/ Word32.fromInt (Array.length (_)))"

code_printing constant length_aa_u_code' ⇀ (SML_imp)
  "(fn/ ()/ =>/ Word32.fromInt (Array.length (Array.sub/ ((fn/ (a,b)/ =>/ a) (_),/
     IntInf.toInt (integer'_of'_nat (_))))))”

code_printing constant nth_raa_i_u64' ⇀ (SML_imp)
  "(fn/ ()/ =>/ Array.sub (Array.sub/ ((fn/ (a,b)/ =>/ a) (_),/
      IntInf.toInt (integer'_of'_nat (_))), Uint64.toFixedInt (_)))"

code_printing constant length_u64_code' ⇀ (SML_imp) "(fn/ ()/ =>/ Uint64.fromFixedInt (Array.length (_)))"

code_printing constant arl_get_u ⇀ (SML) "(fn/ ()/ =>/ Array.sub/ ((fn/ (a,b)/ =>/ a) (_),/ Word32.toInt (_)))"
```

# What is under the carpet? (I)

```
)/ =>/ Array.sub/ ((_),/ Word32.toInt (_)))"

n/ ()/ =>/ Array.sub/ ((_),/ Uint64.toFixedInt (_)))"


Int (_)),/ (_)))"


Int (_)),/ (_)))"

l32.fromInt 2)"

p) "(fn/ ()/ =>/ Word32.fromInt (Array.length (_)))"
```

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informatik

# What is under the carpet? (II)

```
lemma append_aa_hnr[sepref_fr_rules]:
  fixes R ::  ‹'a ⇒ 'b :: {heap, default} ⇒ assn›
  assumes p: ‹is_pure R›
  shows
    ‹(uncurry2 append_el_aa, uncurry2 (RETURN ∘∘∘ append_ll)) ∈
     [λ((l,i),x). i < length l]a (arrayO_assn (arl_assn R))d *a nat_assnk *a Rk → (arrayO_assn (arl_assn R))›
proof -
  obtain R' where R: ‹the_pure R = R'› and R': ‹R = pure R'›
    using p by fastforce
  have [simp]: ‹(∃Ax. arrayO_assn (arl_assn R) a ai * R x r * true * ↑ (x = a ! ba ! b)) =
     (arrayO_assn (arl_assn R) a ai * R (a ! ba ! b) r * true)› for a ai ba b r
    by (auto simp: ex_assn_def)
  show ?thesis — ‹TODO tune proof›
    apply sepref_to_hoare
    apply (sep_auto simp: append_el_aa_def)
     apply (simp add: arrayO_except_assn_def)
     apply (rule sep_auto_is_stupid[OF p])
    apply (sep_auto simp: array_assn_def is_array_def append_ll_def)
    apply (simp add: arrayO_except_assn_array0[symmetric] arrayO_except_assn_def)
    apply (subst_tac (2) i = ba in heap_list_all_nth_remove1)
     apply (solves ‹simp›)
    apply (simp add: array_assn_def is_array_def)
    apply (rule_tac x=‹p[ba := (ab, bc)]› in ent_ex_postI)
    apply (subst_tac (2)xs'=a and ys'=p in heap_list_all_nth_cong)
      apply (solves ‹auto›)[2]
    apply (auto simp: star_aci)
    done
qed
```

# Conclusion

## Concrete outcome

- Watched literals optimisation
- Verified executable SAT solver

## Methodology

- Refinement using the Refinement Framework
- No proof of heuristics (w.r.t. standard)

## Future work

- Restarts (ongoing)
- Use SAT solver in IsaFoR