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Extending an Isabelle Formalisation of CDCL to Optimising CDCL

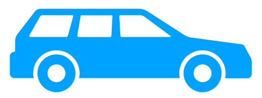
Mathias Fleury

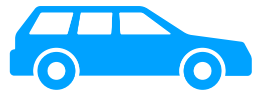
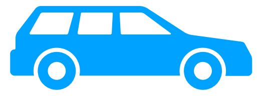
joint work with Christoph Weidenbach

SAARLAND UNIVERSITY
SAARBRÜCKEN GRADUATE SCHOOL OF COMPUTER SCIENCE

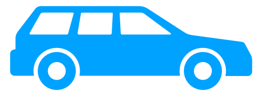
SIC Saarland Informatics Campus

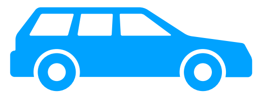
Let’s find a model with minimal weight

 $\vee$ 

 10 $\neg$  4

 20 $\neg$  13

Optimal partial model: 

Optimal total model: $\neg$  

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How reliable is the theory?

Conference version

Branch and Bound for Boolean Optimization and
the Generation of Optimality Certificates
Javier Larrosa, Robert Nieuwenhuis, Albert Oliveras, and Enric Rodríguez-Carbonell (SAT 2009)

A literal l is *true* in I if $l \in I$, *false* in I if $\neg l \in I$, and *undefined* in I otherwise.

A clause set S is true in I if all its clauses are true in I. Then I is called a *model* of S, and we write $I \models S$ (and similarly if a literal or clause is true in I).

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Journal version

A Framework for Certified Boolean Branch-and-Bound Optimization
Javier Larrosa, Robert Nieuwenhuis, Albert Oliveras, and Enric Rodríguez-Carbonell (JAR 2011)

literals of a clause C are false in I. A clause set S is true in I if all its clauses are true in I; if I is also total, then I is called a *total model* of S, and we write $I \models S$.

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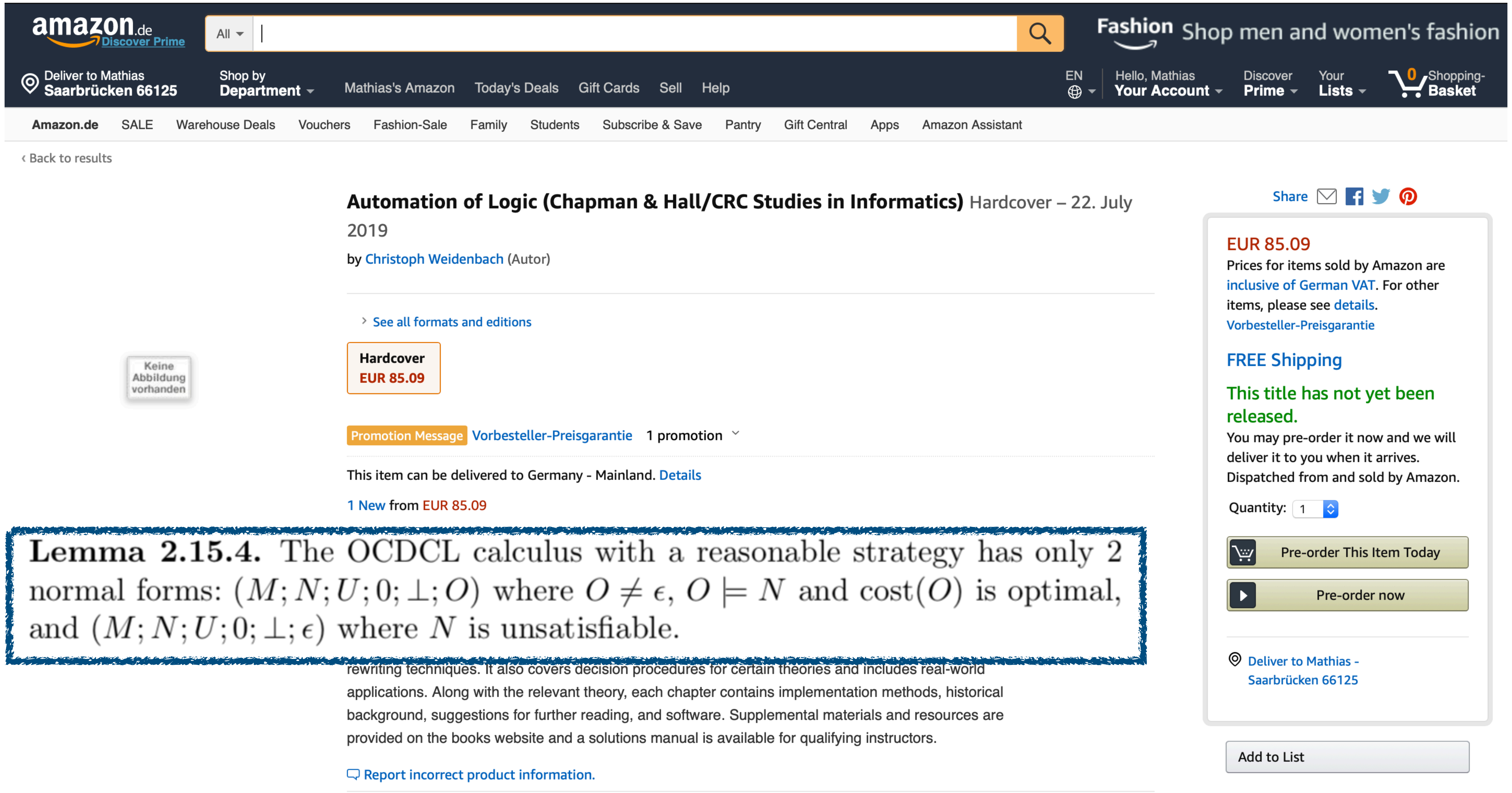
How reliable is the theory?



Lemma 2.15.4. The OCDCL calculus with a reasonable strategy has only 2 normal forms: $(M; N; U; 0; \bot; O)$ where $O \neq \epsilon$, $O \models N$ and $\text{cost}(O)$ is optimal, and $(M; N; U; 0; \bot; \epsilon)$ where N is unsatisfiable.

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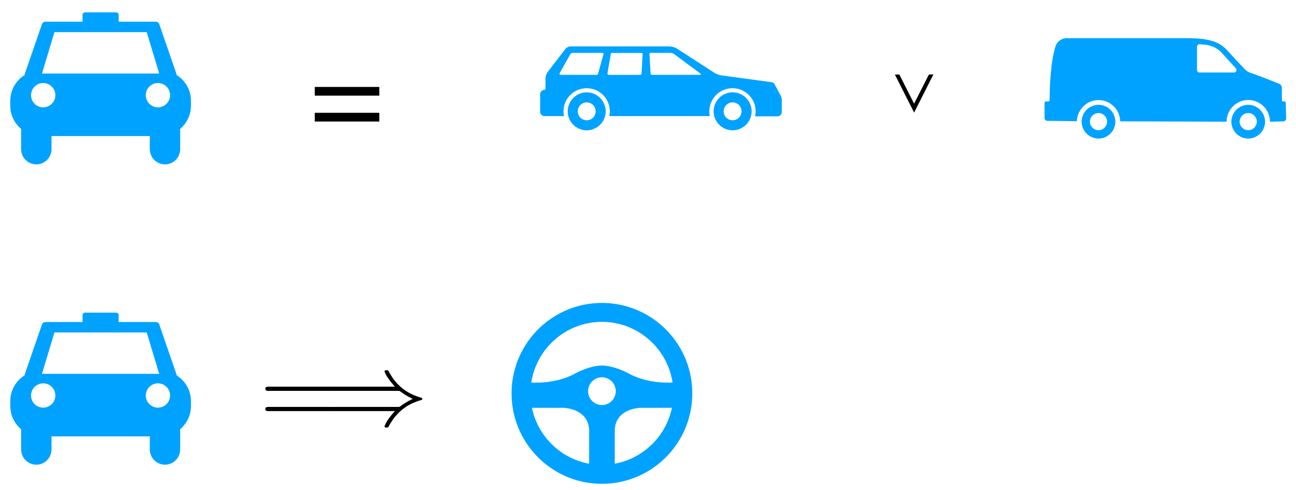
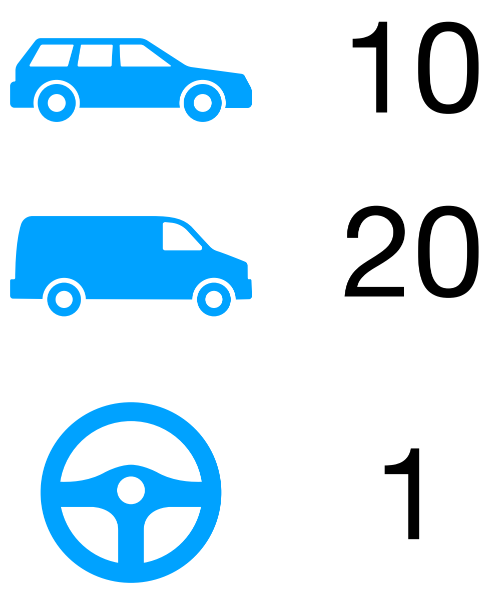
Let's optimise our problem





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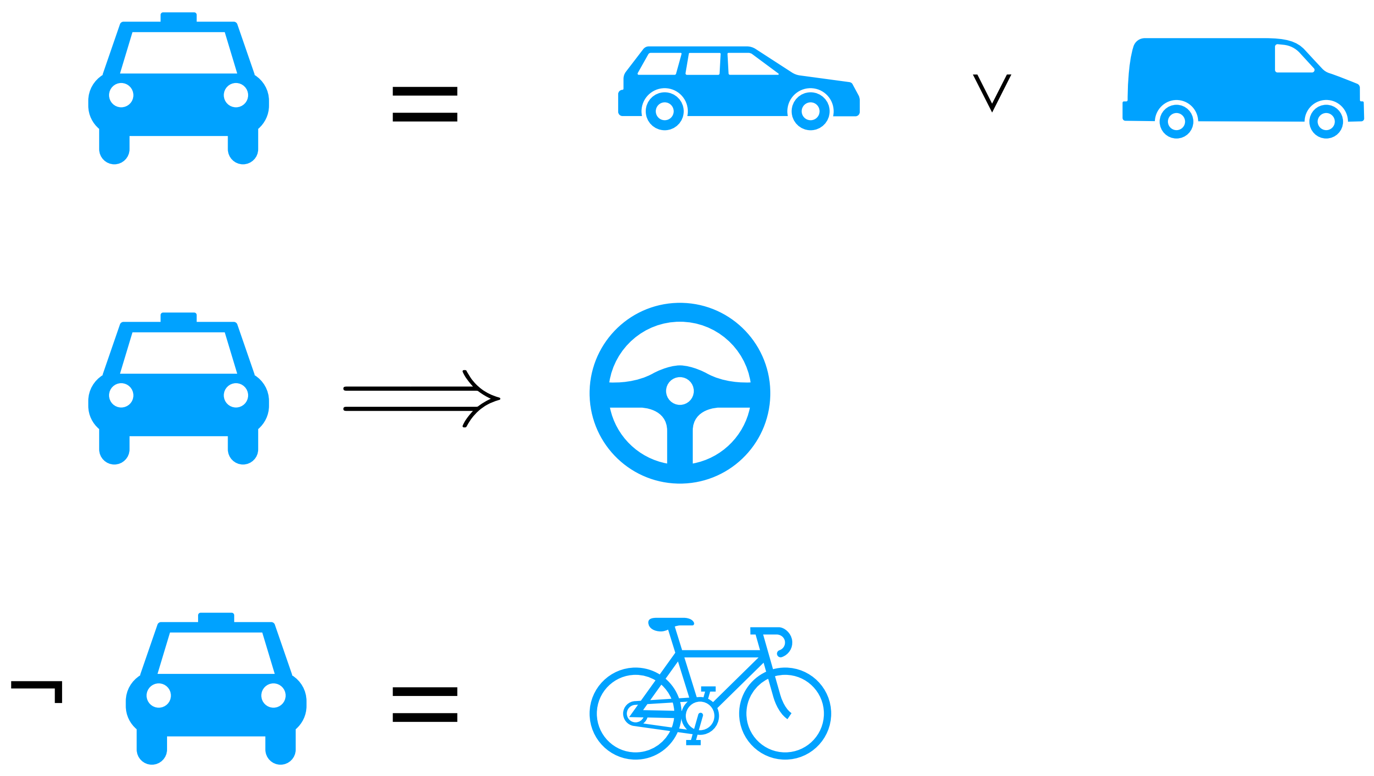
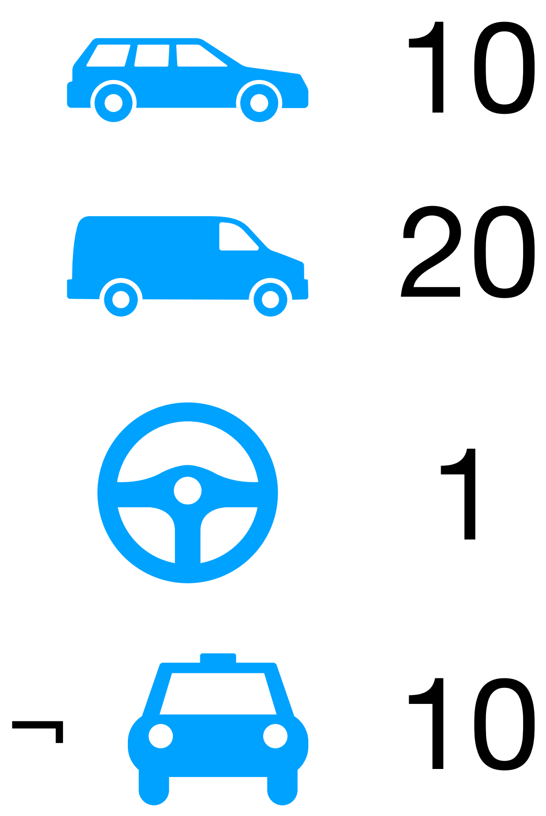
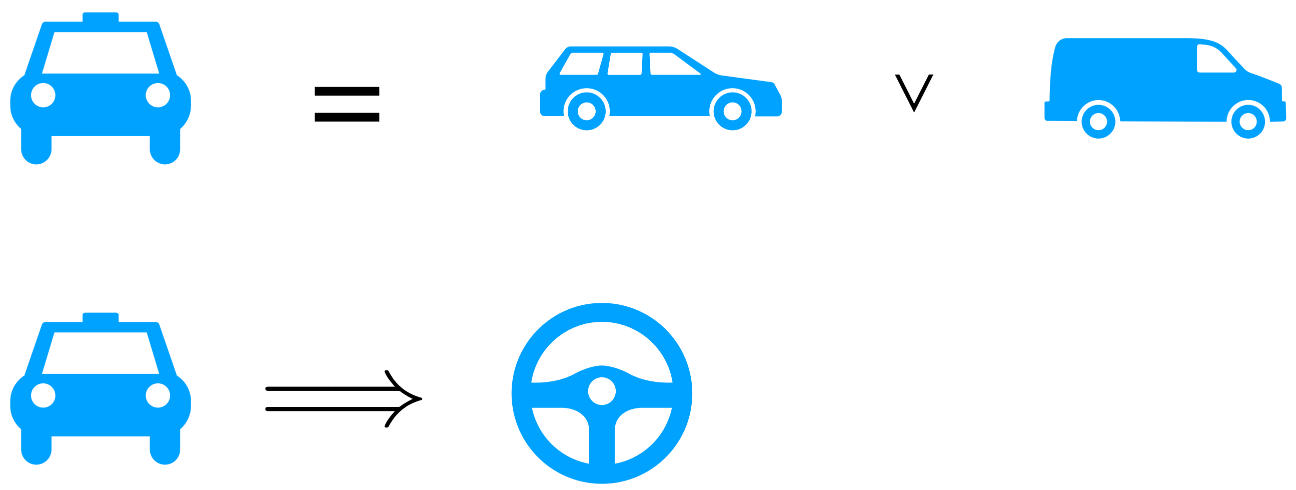


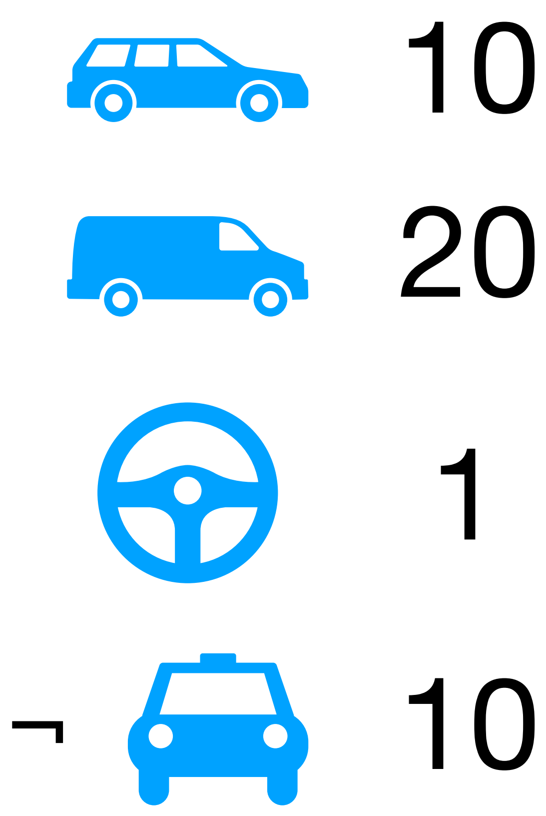


Let's optimise our problem

OCDCL = CDCL + identify better models
+ conflicts based on weights

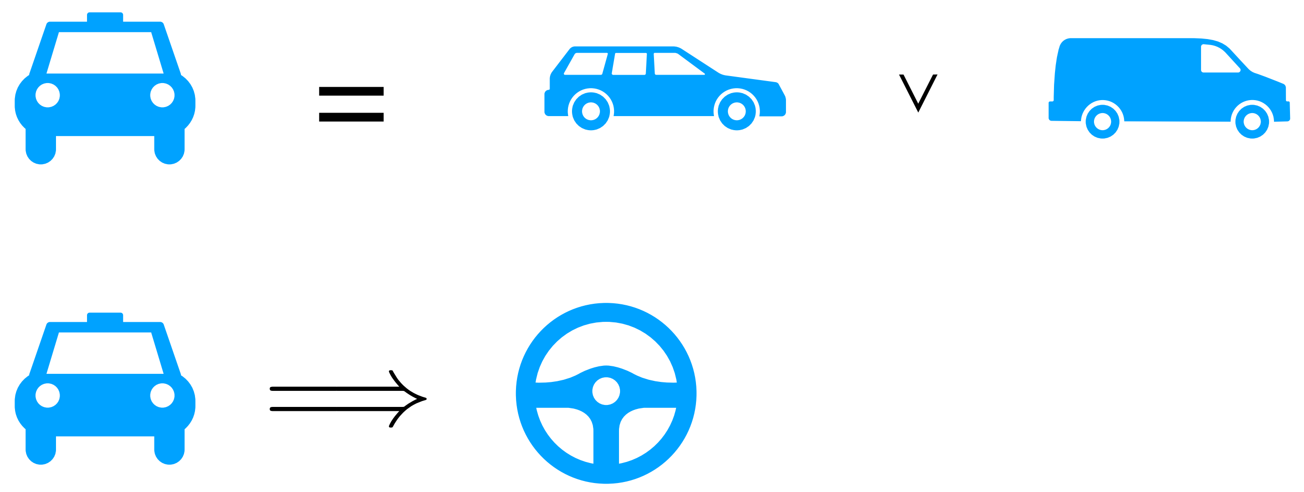
Let’s optimise our problem

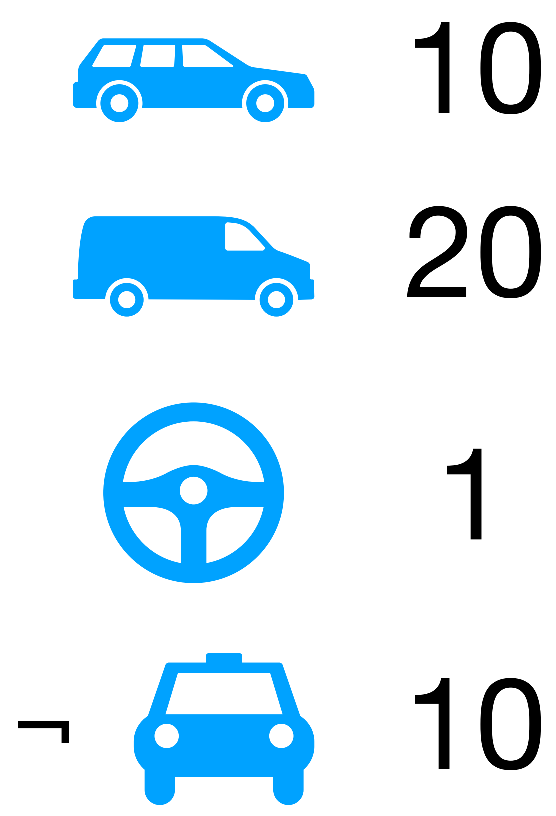




Optimal model

Let’s optimise our problem





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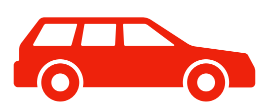

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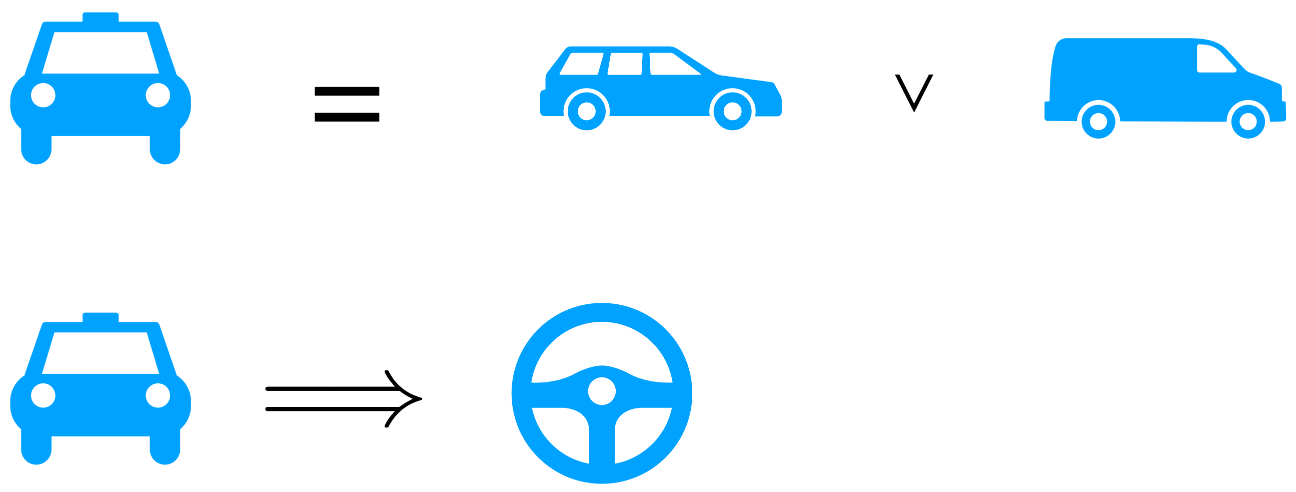

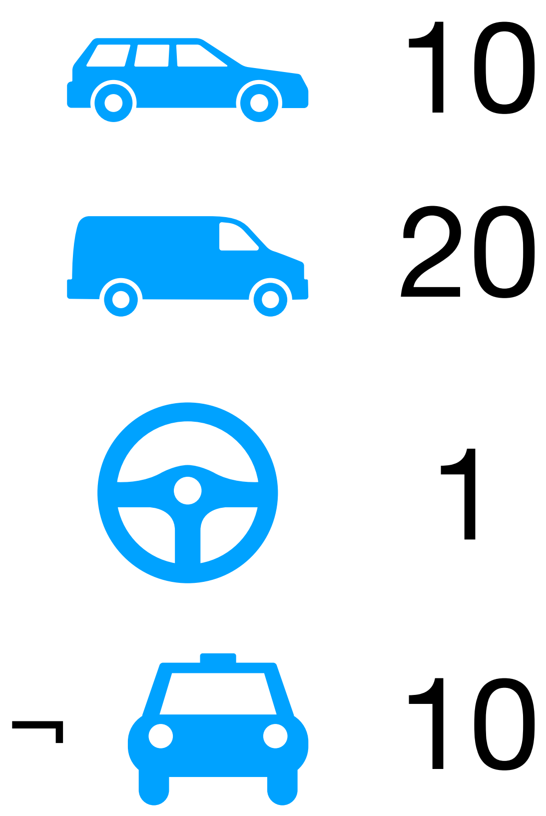

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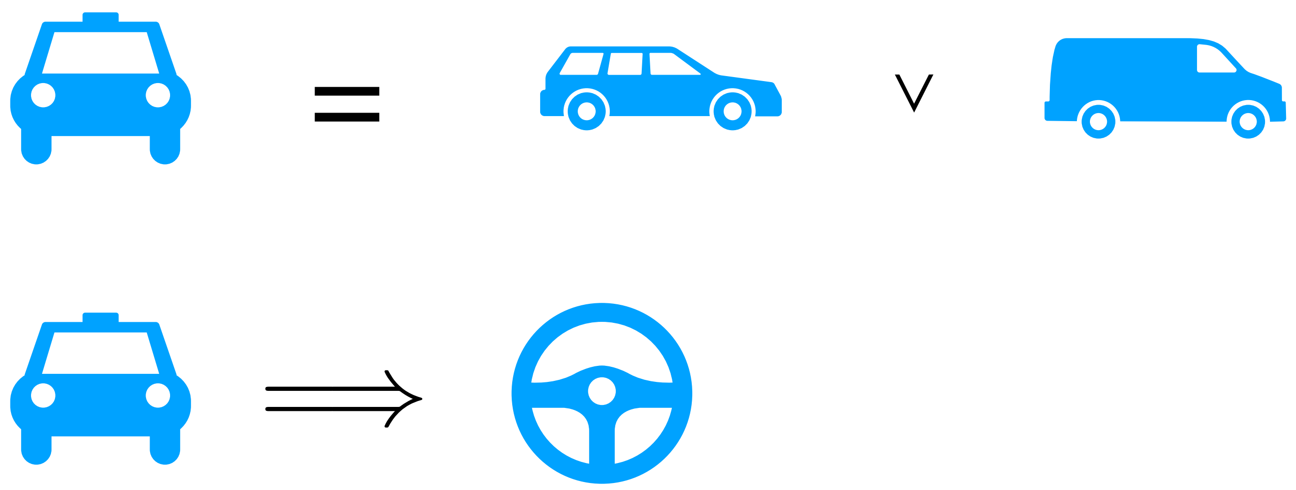

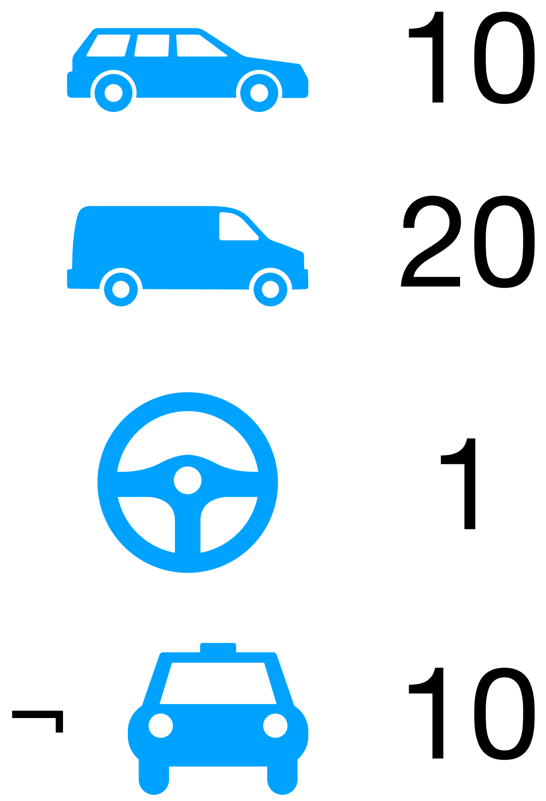

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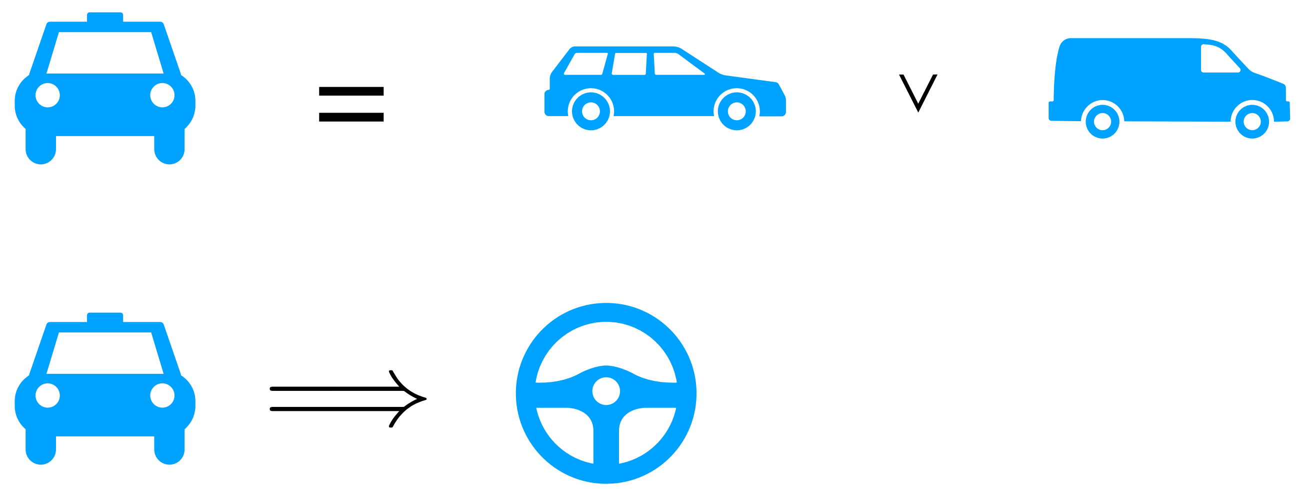

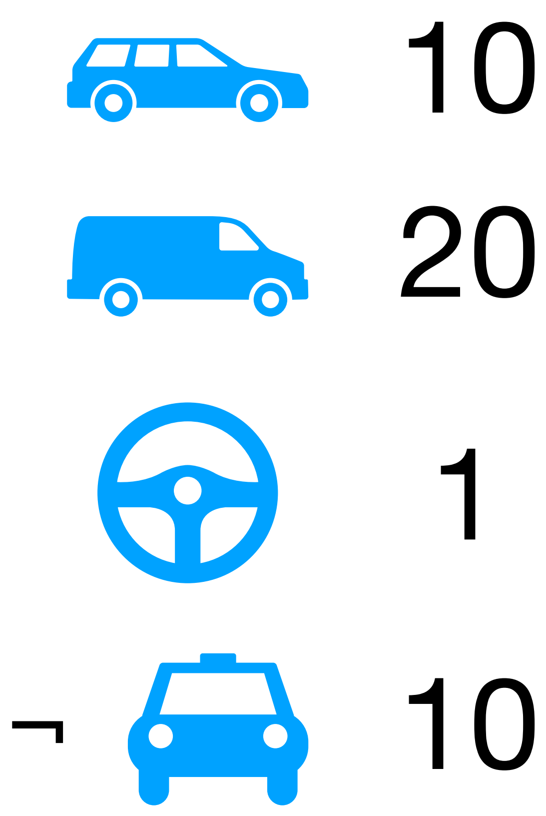

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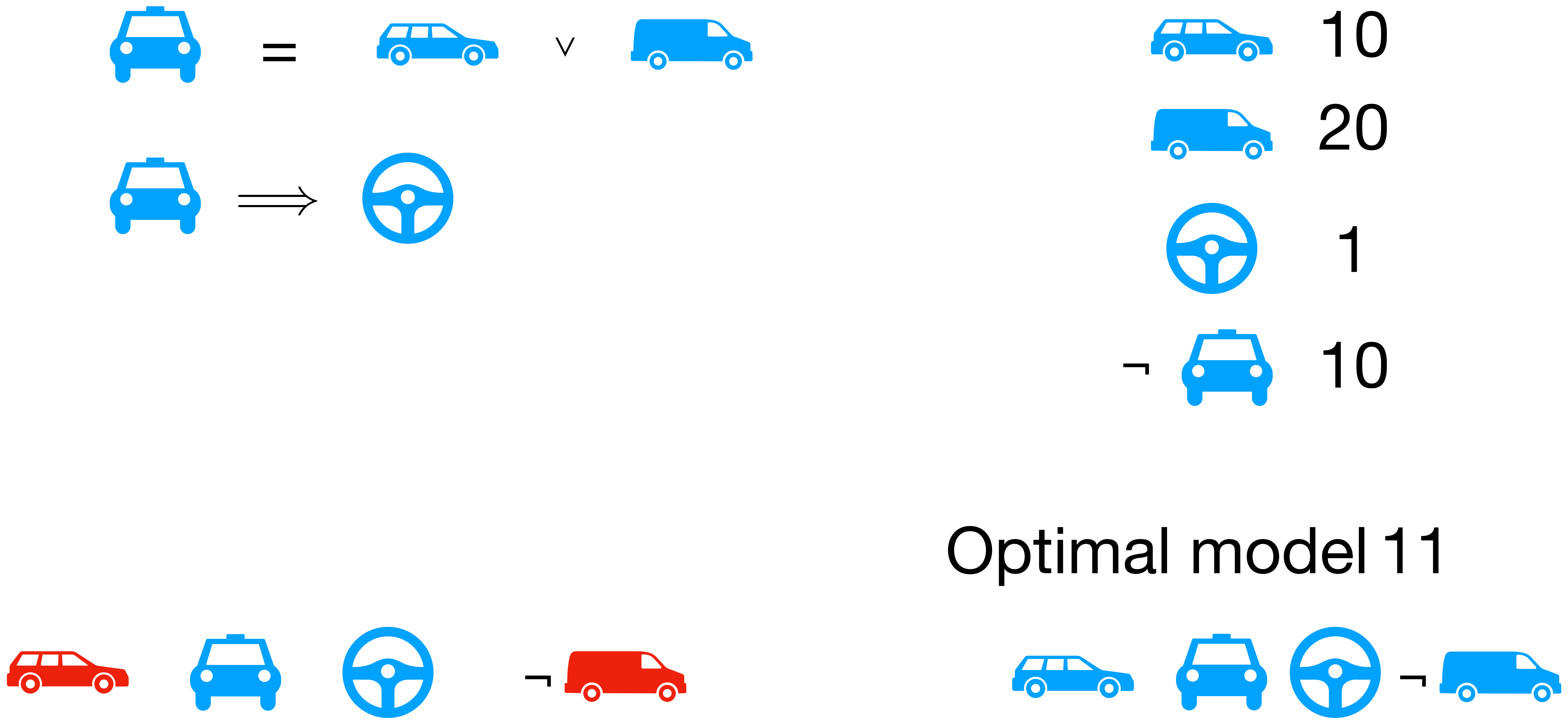

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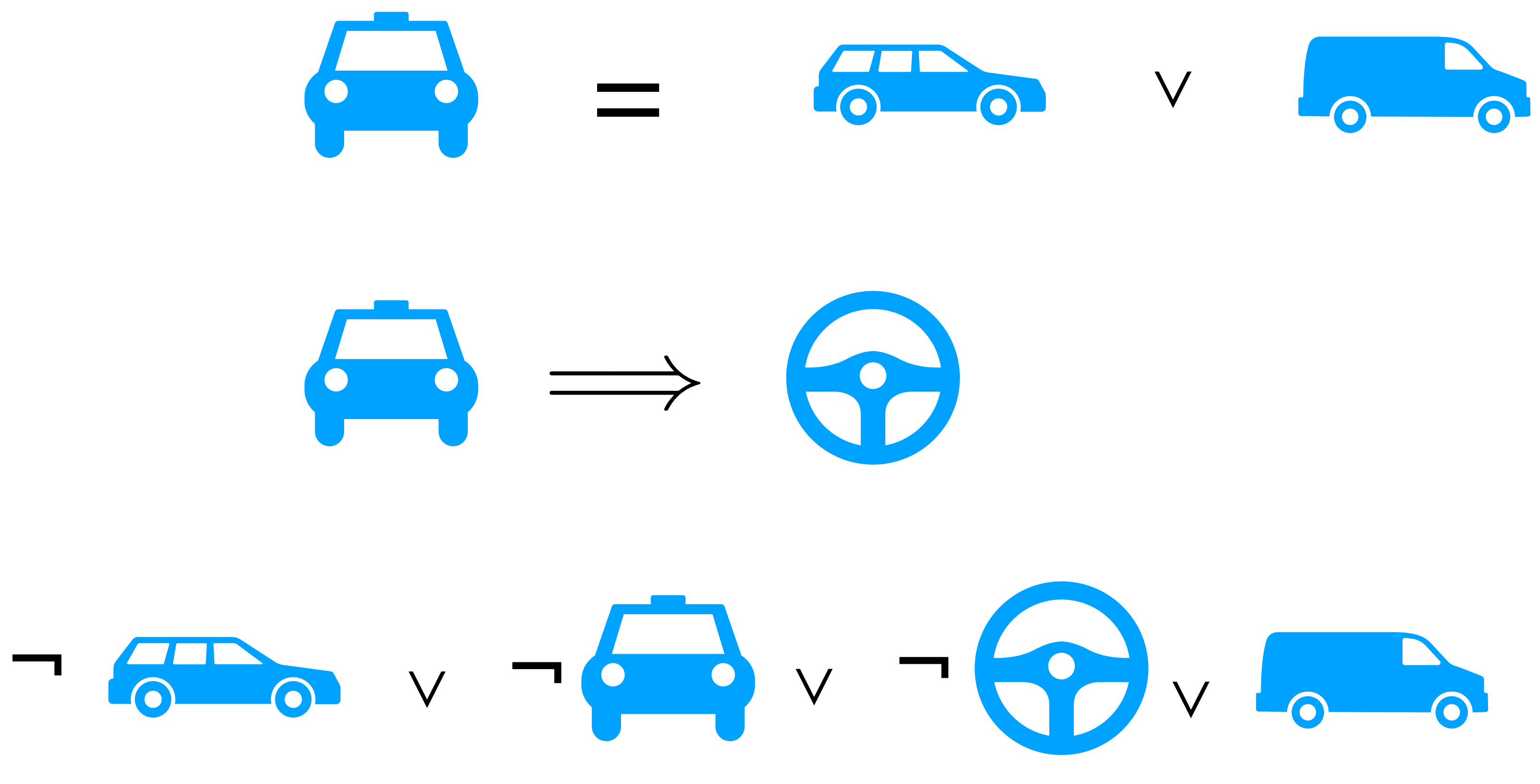
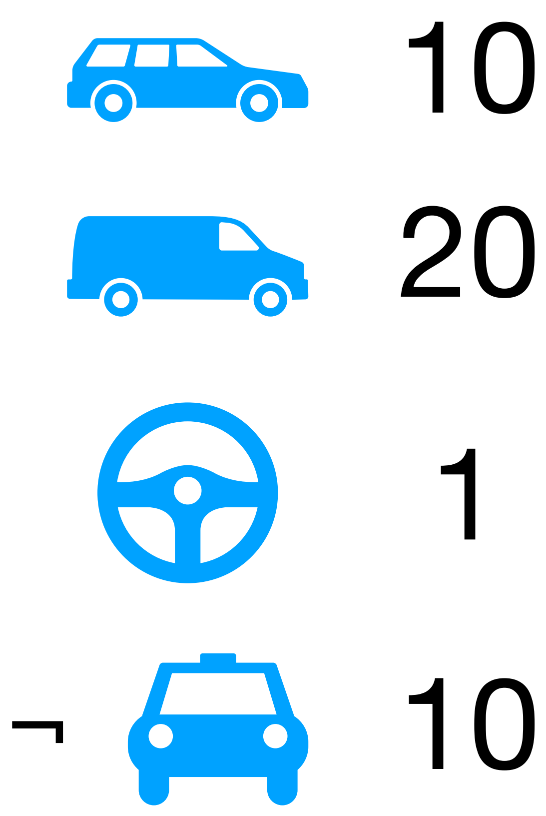
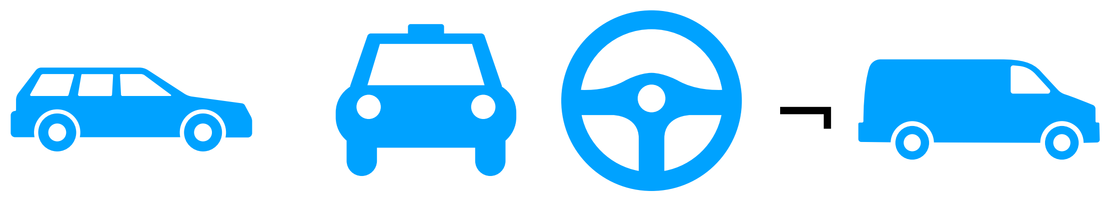
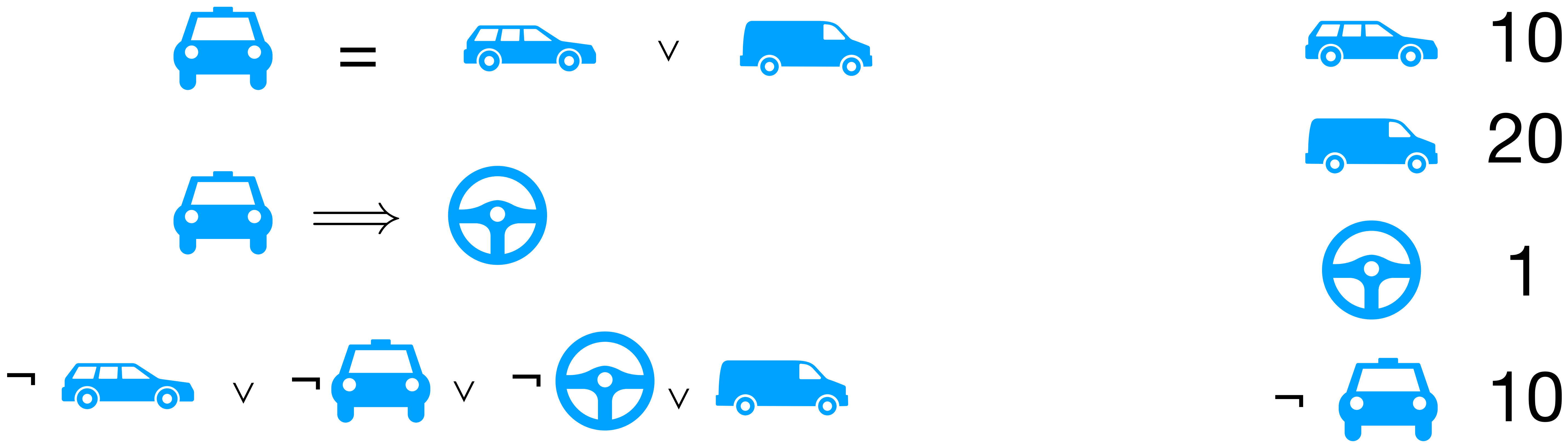




Optimal model 11

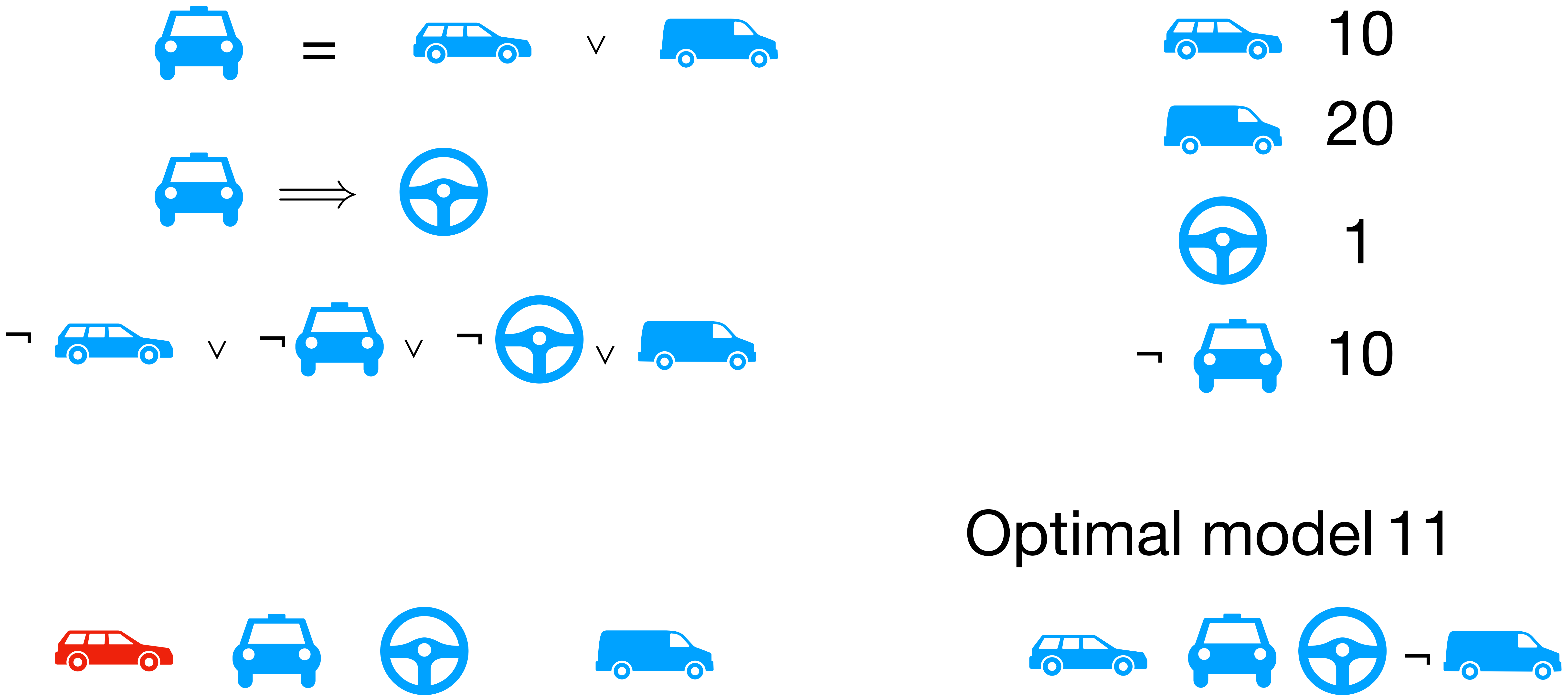


Let’s optimise our problem



Optimal model 11

Let’s optimise our problem



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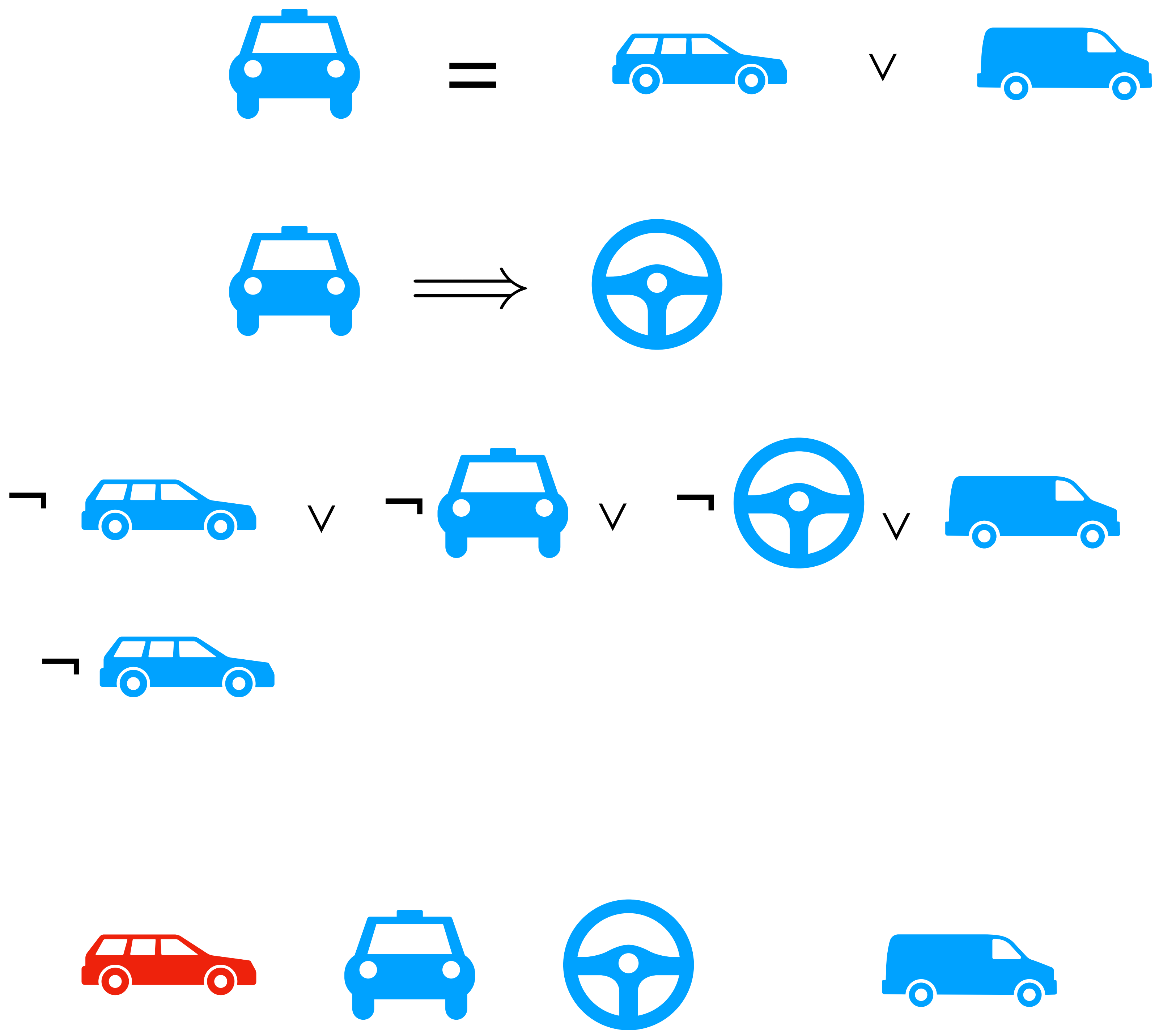

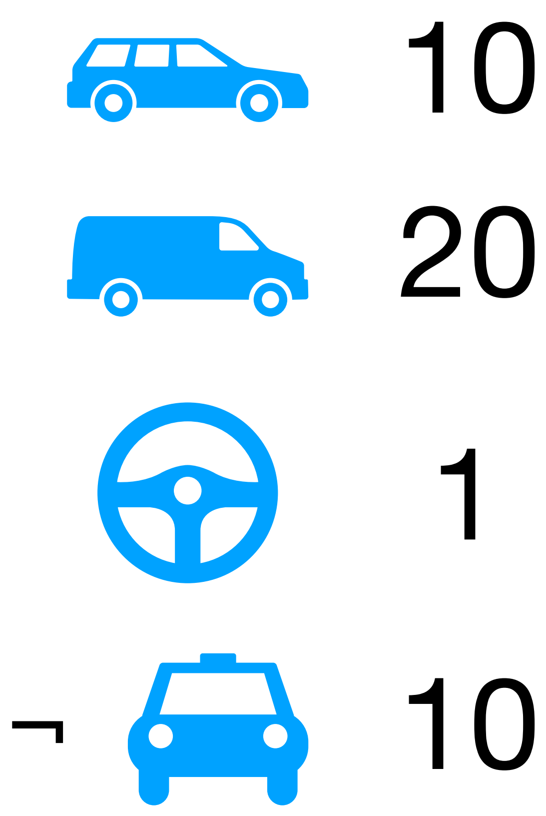

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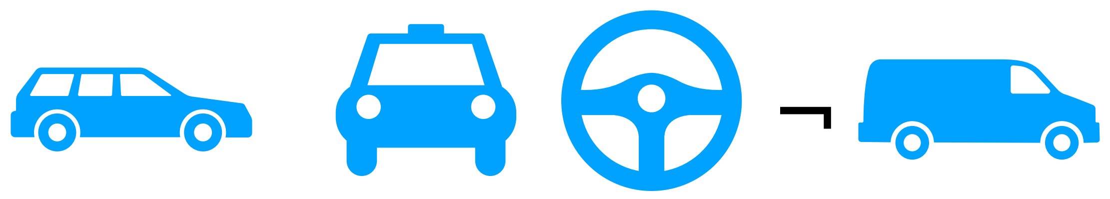

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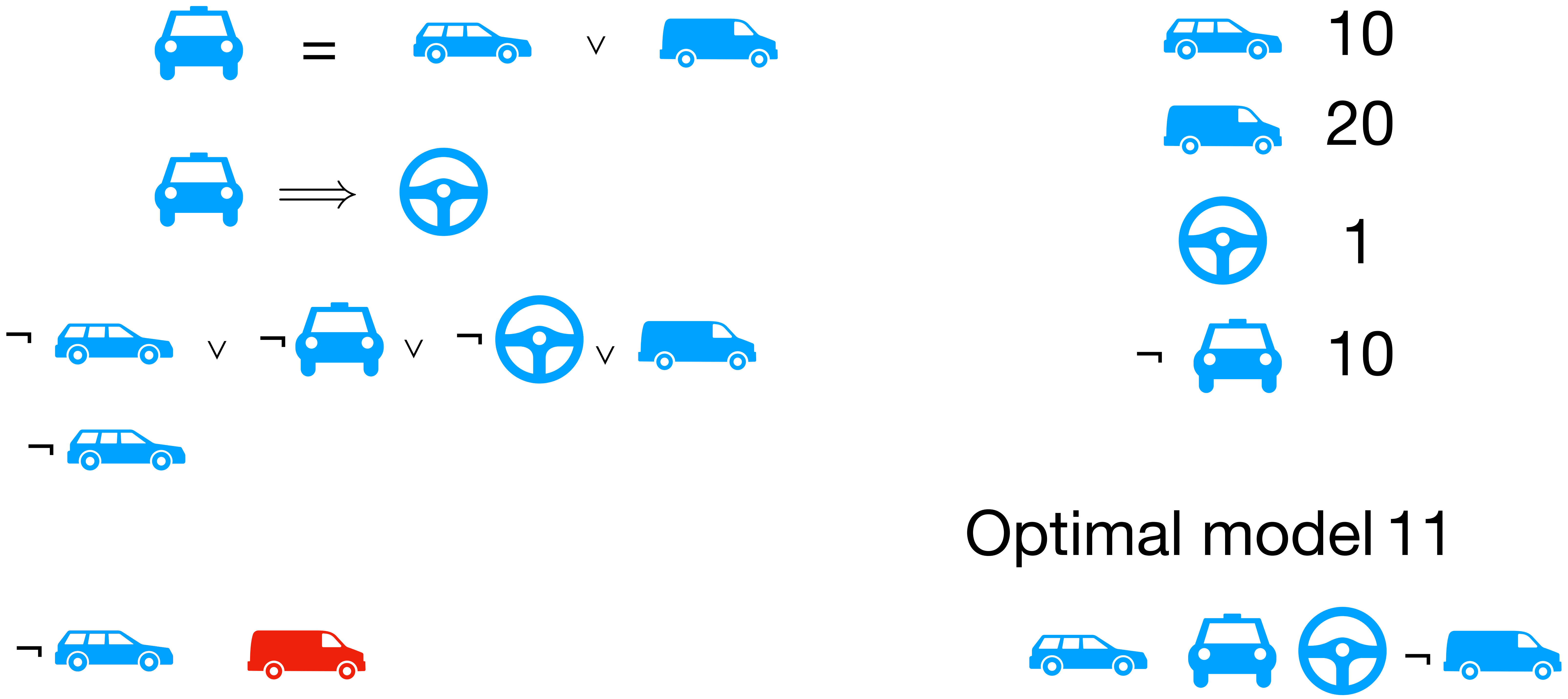

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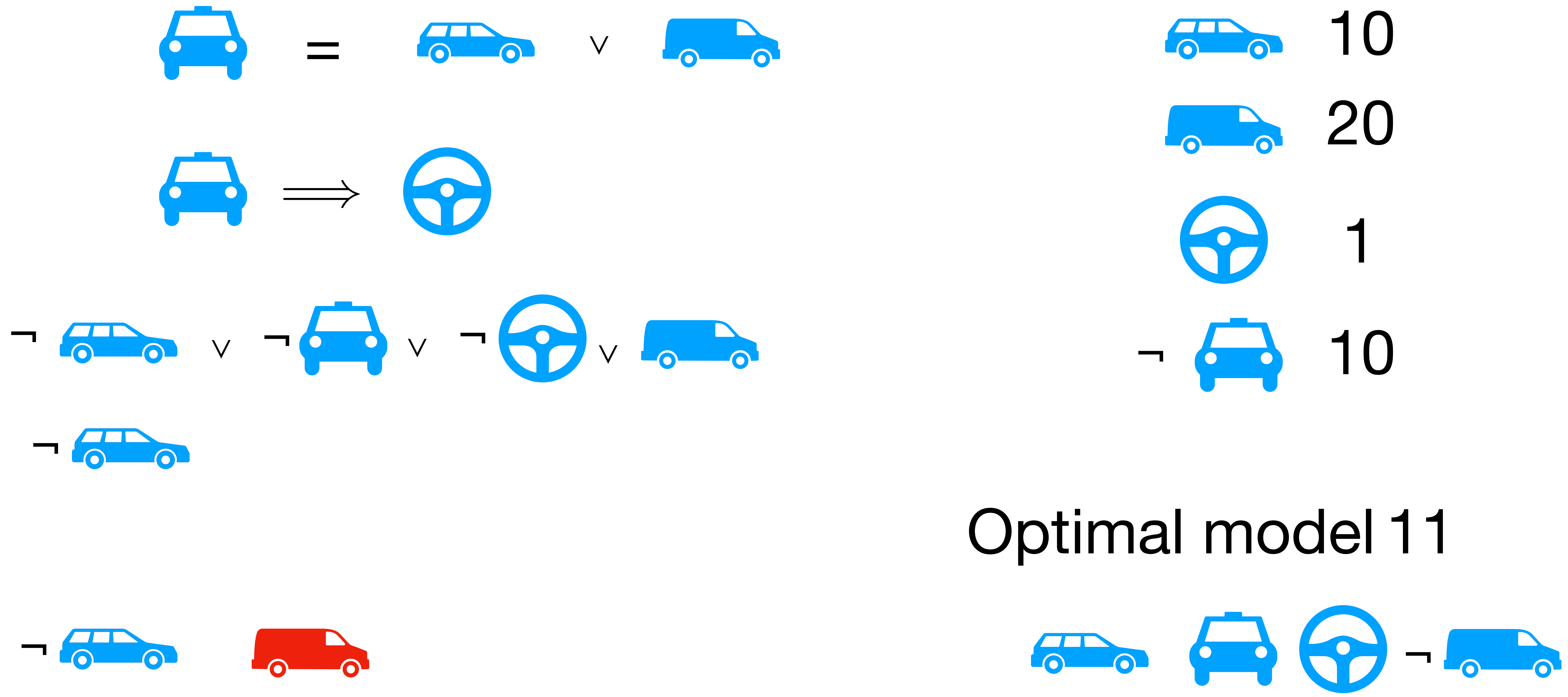

How lazy do you like your formalisation?

Christoph's view:

$OCDCL_W$ = CDCL + improve + conflict rules

copy-paste of proofs

How lazy do you like your formalisation?

Christoph's view:

$OCDCL_W$ = CDCL + improve + conflict rules

copy-paste of proofs

My first idea:

OCDCL = CDCL + improve +
{-M. cost M ≥ min_cost}

reuse CDCL proofs

How lazy do you like your formalisation?

My first idea:

OCDCL = CDCL + improve +
$\{\neg M.\ \text{cost}\ M \geq \text{min\_cost}\}$

reuse CDCL proofs

My second idea:

CDCL_{bnb} = CDCL + improve +
$\mathcal{T}$(min_cost)

reuse CDCL proofs

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How lazy do you like your formalisation?

$CDCL_{bnb}$ = CDCL + improve + $\mathcal{T}$(min_cost)

OCDCL = $CDCL_{bnb}$ where $\mathcal{T}$(min_cost) = {-M. cost M ≥ min_cost}

$OCDCL_W$ = OCDCL + restrictions

Reuse!

$$
\begin{tikzcd}[column sep=huge]
\mathrm{CDCL} \arrow[r, bend right, "\text{properties}"'] & \mathrm{CDCL}_{bnb} \arrow[l, bend right, "\text{can be seen as a fragment of}"'] \arrow[r, bend left, "\substack{\text{by removing the additional component,} \\ \text{is a special case of}}"] & \mathrm{CDCL} \arrow[l, bend left, "\text{definitions and invariants}"]
\end{tikzcd}
$$

Reuse!

can be seen as a fragment of

by removing the additional component,
is a special case of

CDCL

CDCL_{bnb}

CDCL

properties

definitions and invariants

Reuse!

Propagate rule in Isabelle

$C \lor L \in N \Longrightarrow M \models as\ \neg C \Longrightarrow \text{undefined\_lit}\ M\ L \Longrightarrow$
$\quad (M, N) \Rightarrow_{CDCL} (L \# M, N)$

Propagate rule in Isabelle

$C \lor L \in N \Longrightarrow M \models as\ \neg C \Longrightarrow \text{undefined\_lit}\ M\ L \Longrightarrow$
$\quad (M, N, O) \Rightarrow_{CDCLbnb} (L \# M, N, O)$

Reuse!

Propagate rule in Isabelle

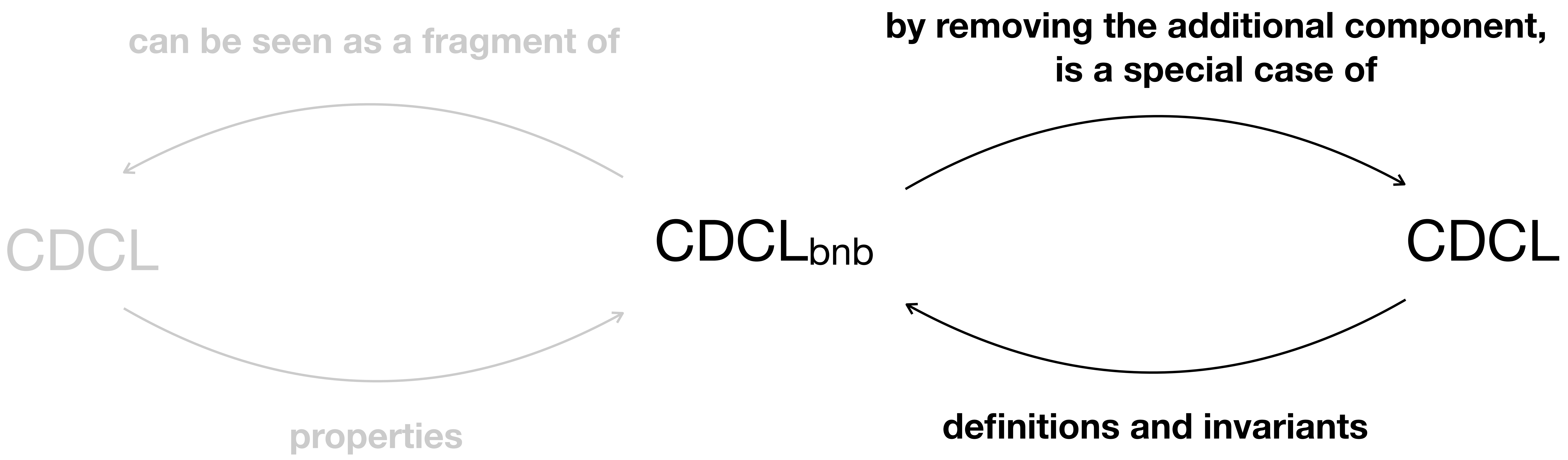
```
C v L ∈ N ⟹ M ⊨as ¬C ⟹ undefined_lit M L ⟹
    (M, N) ⇒CDCL (L # M, N)
```

obtained for free, thanks to abstraction over the state!
also invariants and theorems can be reused

Propagate rule in Isabelle

```
C v L ∈ N ⟹ M ⊨as ¬C ⟹ undefined_lit M L ⟹
    (M, N, O) ⇒CDCLbnb (L # M, N, O)
```

Reuse!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CDCL
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CDCL$_{bnb}$
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definitions and invariants

Reuse!

$$
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\end{tikzcd}
$$

Translate to reuse

Propagate rule

in Isabelle

$C \vee L \in N \Longrightarrow M \models as \neg C \Longrightarrow$ undefined_lit M L $\Longrightarrow$

$(M, N, 0) \Rightarrow_{CDCLbnb} (L \# M, N, 0)$

Propagate rule

in Isabelle

$C \vee L \in N{+}\mathcal{T}(min\_cost) \Longrightarrow M \models as \neg C \Longrightarrow$

undefined_lit M L $\Longrightarrow$

$(M, N + \mathcal{T}(min\_cost), 0) \Rightarrow_{CDCL}$

$(L \# M, N + \mathcal{T}(min\_cost), 0)$

Reuse!

$$\begin{tikzcd}[column sep=huge]
\text{CDCL} \arrow[r, bend right, "\text{properties}"'] & \text{CDCL}_{\text{bnb}} \arrow[l, bend right, "\text{can be seen as a fragment of}"'] \arrow[r, bend left, "\substack{\text{by removing the additional component,} \\ \text{is a special case of}}"] & \text{CDCL} \arrow[l, bend left, "\text{definitions and invariants}"]
\end{tikzcd}$$

Reuse in practise!

ignore the additional component

CDCL_{bnb} = CDCL + improve + $\mathcal{T}$(min_cost)

Inherited:

Definitions (for free)

Reuse in practise!

no strategy
but terminating

well-founded
for most applications

$\mathrm{CDCL}_{\mathrm{bnb}}$ = CDCL + improve +
$\mathcal{T}$(min_cost)

Inherited:

Termination (for free) Definitions (for free)

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Reuse in practise!

strategy $\Longrightarrow$ **strategy invariants holds**

improve can always be applied on total models

CDCL_{bnb} = CDCL + improve + $\mathcal{T}$(min_cost)

Inherited:

Termination (for free)   Definitions (for free)

Ends with an unsat set (nearly for free)

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Reuse in practise!

CDCL_{bnb} does not know anything about what is optimised!

Inherited:

Termination (for free)

Definitions (for free)

Ends with an unsat set (nearly for free)

Why does it work?

OCDCL = CDCL_{bnb} where

$\mathcal{T}(\text{min\_cost}) = \{-M.\ \text{cost } M \geq \text{min\_cost}\}$

Why does it work?

OCDCL = CDCL_{bnb} where
$\mathcal{T}$(min_cost) = {-M. cost M ≥ min_cost}

Lemma

If I is a total model of N with cost < min_cost, then I is a model of N+$\mathcal{T}$(min_cost)

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Why does it work?

Lemma

If I is a total model of N with cost < min_cost, then I is a model of N+$\mathcal{T}$(min_cost)

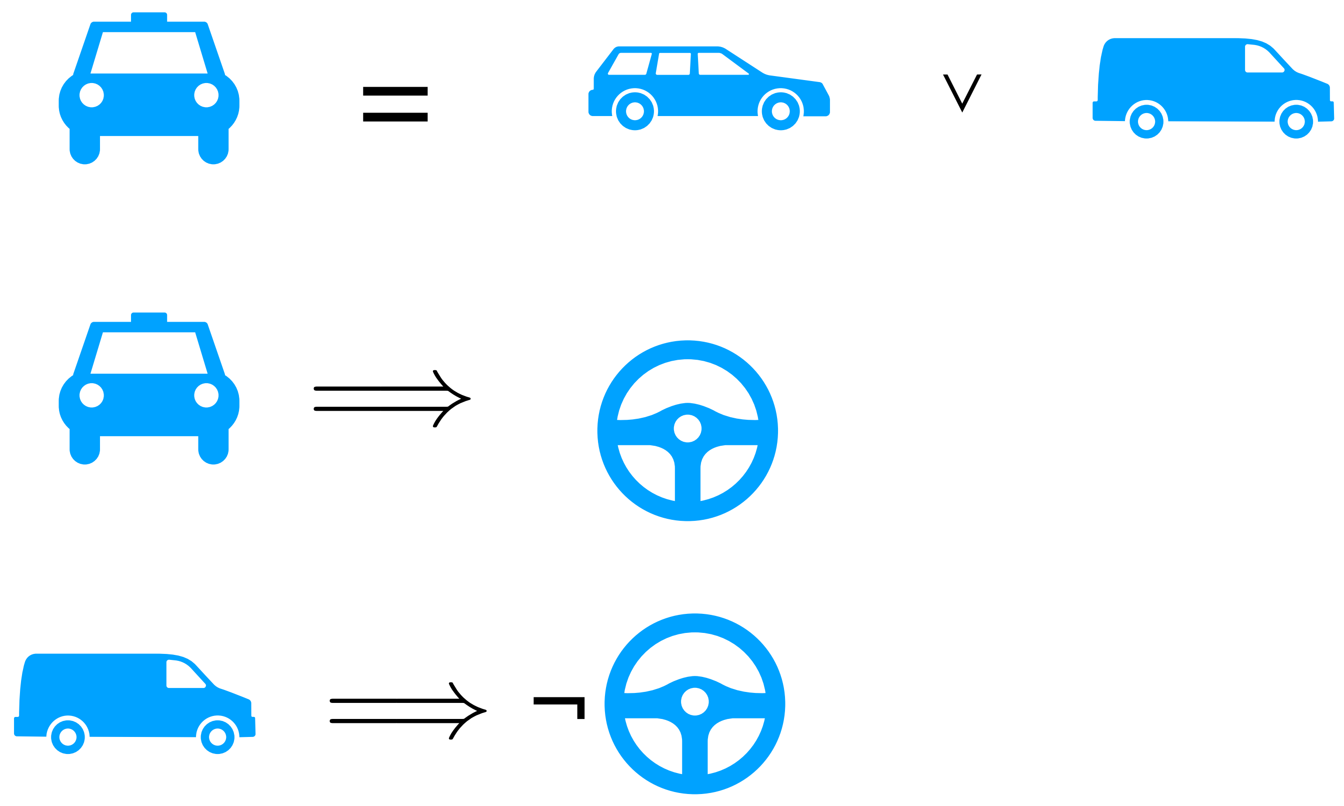
Fails for partial models!

How lazy do you like your formalisation?

make sure that the rules on paper and in Isabelle are the same

$\mathrm{OCDCL}_W = \mathrm{OCDCL} + \text{restrictions}$

Another application: Dead features



Can every option be true?

How lazy do you like your formalisation?

Christoph's view:

CDCLcm_W = CDCL + improve + conflict rules

copy-paste of proofs

My idea:

$\text{CDCLcm} = \text{CDCL}_{\text{bnb}}$ where

$\mathcal{T}$(models_founds) = {-M. there is a model with more trues in models_founds}

How lazy do you like your formalisation?

How lazy do you like your formalisation?

	Lines of codes (kloc)
CDCL_{bnb}	2,0
OCDCL	1,8
OCDCL_{W}	1,0
Partial Encoding	1,2

(a lot of boilerplate)

How lazy do you like your formalisation?

	Lines of codes (kloc)
CDCL_{bnb}	2,0
OCDCL	1,8
OCDCL_{W}	1,0
Partial Encoding	1,2
MaxSAT	0,4

(a lot of boilerplate)

can solve

Conclusion

Concrete outcome

- CDCL with branch and bound
- Via an encoding, also partial optimal models

Methodology

- Locales, locales, locales
- Be lazy!

Future work

- CDCL($\mathcal{T}$)

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Conclusion

Concrete outcome

- CDCL with branch and bound

OCDCL = CDCL_{bnb} where

$\mathcal{T}$(min_cost) = {-M. cost M ≥ min_cost}

OCDCL = CDCL_{bnb} where

$\mathcal{T}$(min_cost) = {-D. {M. cost M ≥ min_cost} ⊨ D}

Future work

- CDCL($\mathcal{T}$)

Conclusion

Concrete outcome

- CDCL with branch and bound
- Via an encoding, also partial optimal models

Methodology

- Locales, locales, locales
- Be lazy!

Future work

- CDCL($\mathcal{T}$)

Conclusion: How about CDCL($\mathcal{T}$)?

But isn't CDCL($\mathcal{T}$) exactly:

CDCL$_{bnb}$ where
$\mathcal{T}$ = {clauses entailed theory}

Not exactly, because the wrong conflict clause (negation of the trail) is used

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Translate to reuse

Theory propagation

Propagate rule

in Isabelle

C ∨ L ∈ N + $\boxed{\mathcal{T}\text{(min\_cost)}}$ ⟹ M ⊨as ¬C ⟹
undefined_lit M L ⟹
(M, N + $\mathcal{T}$(min_cost), 0) ⇒CDCLbnb
(L # M, N + $\mathcal{T}$(min_cost), 0)